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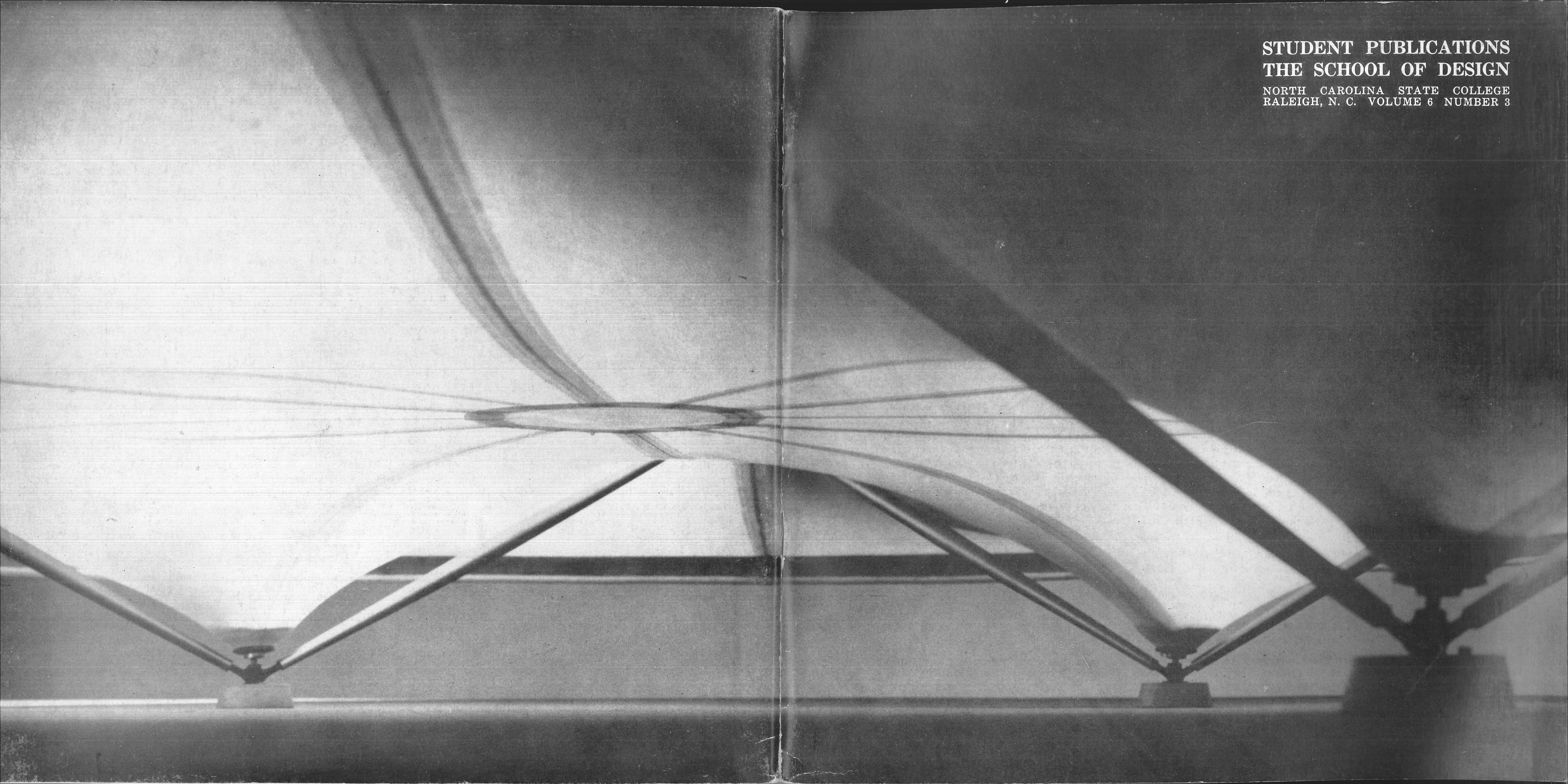

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The real problems of a revolution begin when the battle is won. The slogans and catch-phrases which serve their purpose well when the attack is on, prove, as a rule, poor guides to action when the responsibility for solid, evolutionary progress is assumed. Now that the architectural revolution of the last hundred years is fully accepted and widely implemented, the course of architectural development in the second half of the twentieth century is vague and uncertain. A refinement of now-existing forms and ideas is valuable to a point, but there is a general consensus that the work of the first half of this century left many problems unsolved and many others unrecognized.

There are perhaps three major avenues of further development which have been proposed: a more intensive investigation and exploitation of the structural potentialities of modern materials and the formal potentialities of these structures; a shift of emphasis away from the problems of the individual building to those of the total environment, particularly those of the urban complex; and finally, a re-integration of contemporary architecture with our cultural tradition, and a wider acceptance of human as well as formal needs. These three directions are not necessarily mutually compatible, but they are undoubtedly the keys to a vigorous future for architecture. In this issue we are primarily concerned with the first approach. The succeeding two issues will develop more extensively the other two.

The integrations of structure and form is one of the most difficult problems of the architect, and when solved, one of his highest achievements. It represents a synthesis of human purpose and natural necessity. There are understandably few examples of such an integration. It is most widely found in peasant cultures where the limitations of nature are so great that an arbitrary form is out of the question; and in engineering structures, both ancient and modern, where the purpose is sufficiently straightforward not to interfere with a logical use of materials. The Gothic cathedrals represent, to the present, the fullest achievement of this synthesis. Recent years have seen the attempt at a new synthesis in terms of modern materials. There is the well-known work of those who have been labeled the "creative engineers": Nervi, Torroja, and Candela. Their work is highly significant and often beautiful. But the final synthesis must still await those men, usually called 'architects', whose outlook and training render them capable of dealing creatively with both purpose and necessity.

That this will require a much deeper understanding of structural behavior, structural potentialities, and constructional limitations, is obvious. Many Ammerican architects are today using more daring structures, but more often than not the structure is essentially a visual, rather than a physically necessary, form. It can be built only with undue effort and cost, and often the very visual effect the architect intended must be partially destroyed in the interest of safety. On the other hand, American engineers are limited in their structural daring by the difficulties of mathematical analysis. No facilities exist in this country for the empirical testing of large scale models comparable to those of Torroja in Spain, or Oberti in Italy.

The present article, then, while presenting an original membranal structure, is, incidentally, an outline of the problems to be solved by any structural concept, and of the methods that can be employed in analysing and realizing such a concept.

THE EDITORS

TWO TYPES OF MEMBRANAL STRUCTURES

HORACIO CAMINOS
ATILIO GALLO
GIUSEPPE GUARNIERI

CONTENTS

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For their contributions the authors are greatly indebted to: Research and Design: WAYNE TAYLOR *(Student, School of Design)*. Previous studies. ROY GUSSOW *(Associate Professor of Design)*. Wind Tunnel Tests: ROBERT M. PINKERTON *(Professor of Aeronautical Engineering)*, JOHN J. CLEARY *(Assistant Professor of Aeronautical Engineering)*. Research in Models: CHARLES H. KAHN *(Instructor in Civil Engineering)*. Photographs: LANDIS S. BENNETT, RALPH A. MILLS *(Visual Aids)*. Drawings and Models: JAMES C. PYRON, MAX ISLEY, IGNACIO M. ZUBIZARRETA, EDWIN F. HARRIS, JOSEPH HOSKINS *(Students, School of Design)*, ROBERT P. BURNS, CHARLES D. DAVIDSON, WILLIAM W. DODGE III, BERTRAM ELLENTUCK, LUCIUS R. EVANS, WARREN E. HARGETT, IRWIN E. JONES, FREDERICK W. KOONTZ, GORDON B. SHERMAN, NOMERIANO S. SON, JOHN T. WALL, JAMES R. WASHBURN, CHARLES E. WOODALL, NORMAN L. ZIMMERMAN, *(5th Year Students, Class 1957, School of Design)*.
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INTRODUCTION

Two types of membranal structures are presented in this article. An approach to a general expression of equilibrium for membranes with central vertical axis is also published, as far as is known, for the first time.[1]

Both types of structures are based upon the principle of stretching a membrane by means of vertical forces acting in opposite directions. Only a few particular cases developed are discussed and illustrated. Nevertheless, applying the same principle, infinite variations can be obtained. Some of these other possibilities are also shown here, although very schematically.

The two types presented are:

A. Membranes primarily in pure tension, suitable for perishable materials like canvas, plastics, fabrics, etc.; or more durable such as sheet metals; or, finally, a combination of a structural net (wires, cables, rods) and a non-structural skin.

B. Rigid shells originated from the former membranes by freezing the shape and reversing or removing the generative forces. Reinforced concrete and again, sheet metals are suitable materials for this type.

In the preparation of this article, its accessibility to students was kept in mind. Hence, the inclusion of elementary considerations of equilibrium and the sequence followed in its presentation.

It happens that this sequence is somewhat the same as the one followed in the process of design, namely: the fundamental assumption of structural behavior on which the structures are based; the resulting structures; and an approach to the stress analysis. This presentation is rather general and is by no means completed since analytical studies and experimental work are in progress. A second article has been planned concerning mainly the problems of construction and stress analysis.

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[1] These studies were initiated in Raleigh, North Carolina, in the winter of 1953. Caminos developed the structures in consultation with Gallo. In the fall of 1956, Guarnieri developed the studies on the membranal equilibrium.

I ELEMENTAL CONCEPTS OF EQUILIBRIUM

FORMS OF EQUILIBRIUM OF A FLEXIBLE THREAD

GENERAL CONDITION: Assume a flexible thread supported in two points *A* and *B* at the same level. (fig. 1)

FORM OF EQUILIBRIUM WITH UNIFORM LOAD DISTRIBUTION ALONG THE CHORD. The thread is uniformly loaded with vertical loads along the chord A-B (fig. 2). The thread is deformed by action of the load and takes a position of equilibrium that is a *parabola* ABC.

This curve is a function only of the load distribution if the strain elongations of the thread are neglected. If the distribution of the loads is uniform, this equilibrium curve shall be always a parabolic curve.

On points of support A and B there are two reactions that jointly with the load constitute an equilibrate system. Knowing the curve ABC it is possible to replace the former thread by another of different material following the same curve ABC. In that case the equilibrium between loads and reactions remains unaltered. The total internal stresses for each section of both threads will be the same. In summary: the equilibrium is independent of the material if elongations are negligible.[2] Thus, the parabola ABC is the curve of equilibrium for uniform load distribution along the chord.

It is noticed, finally, that the stresses in the thread are all tensile and can be calculated by analytical or graphic methods. Stresses change along the thread: at the central point they are minimum, at the supports, maximum.

FORM OF EQUILIBRIUM WITH UNIFORM LOAD DISTRIBUTION ALONG THE THREAD. The same considerations stated above can be extended to the case of load uniformly distributed along the thread (fig. 3). The curve of equilibrium will be a *catenary*. Stresses again are all tensile and vary in different points of the thread.

When the rise *R* of the catenary is small, it can be seen that there is not too much difference between uniform load along the chord and along the curve. Therefore, for practical purposes, the parabola can be adopted as the curve of equilibrium. (The equation of the parabola is a much simpler expression than that of the catenary).

FORM OF EQUILIBRIUM WITH CONCENTRATED LOADS. Concentrated loads will deform the thread into a polygonal line with points of inflexion where loads are applied. This line of equilibrium is the *funicular polygon* passing through the supports A-B. (Figure 4).

SUMMARY. In all the cases stated above the natural form is equivalent to the funicular polygon of loads. The internal stresses in every point of the thread and the reactions can be obtained analytically or graphically through the force diagram.

[2] Because the system is statically determined, the equilibrium conditions are independent of the material as well as of the law of variations of the sections.

EQUILIBRIUM OF A RIGID THREAD

OBSERVATIONS. In all the cases previously considered the thread acts in pure tension. If the thread is frozen in such a way that its shape cannot be changed, new systems of loads can be applied with the following consequences:

A. *New system of loads identical to the one that has produced the natural form when the thread was flexible:* No difference between the former and the new internal stresses.

B. *New system of loads proportional to the former system with a coefficient (K):* The form is not affected. The internal stresses of the former and the new system are in the relation *K*.

C. *New system of loads different from the former system:* The form does not accommodate to the new system. There are bending moments.

D. *New system of loads identical to the former but of opposite sign:* The form is not affected. The internal stresses and reactions retain the same value but their signs are reversed. Tensile stresses become compressive stresses. These considerations are valid for the three cases illustrated above and in general for any other case.

CONCLUSIONS

A. For a particular load situation there is only one natural form that produces pure compression or pure tension.

B. For a particular form there is only one system of loads that produces pure compression or pure tension. The same can be stated for any other proportional load system. In other terms: the new system is equal to the former system multiplied by *K*.

C. If a form is known, it is possible to find a system of loads that produces only tensions and compressions by means of a diagram of forces with its respective vectors parallel to the tangents of the form. To multiply the load by a value *K* is equivalent to changing the value *H*, where *H* is the polar distance on the diagram of forces.

D. The forms of equilibrium are reversible. If the direction of the loads is reversed without altering the form, the equilibrium still holds; but the direction of the reactions, A and B, reverse, and the tensile stresses become compressive stresses.

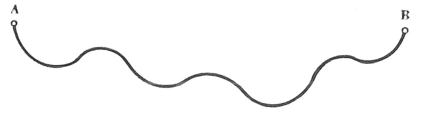
FORMS OF EQUILIBRIUM OF A MEMBRANE

OBSERVATIONS. The preceding conclusions refer to the equilibrium of forces in a plane. If a generalization is made of forces in the space, the *curves of equilibrium* become *surfaces of equilibrium.* To extend the generalization, the flexible thread can be substituted by a flexible membrane.

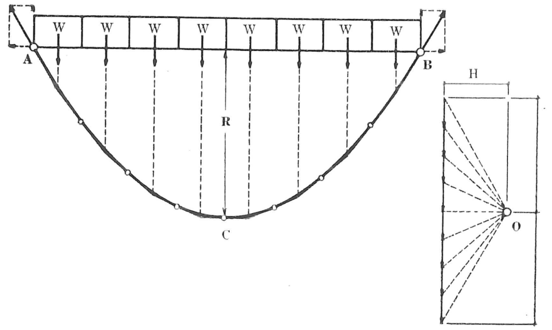
A *membrane* is a resistant element whose thickness is negligible in relation to its length and width: in a proportion of 1:1000 or less. The following assumptions are made:

A. That the membrane resists only tensile stresses.

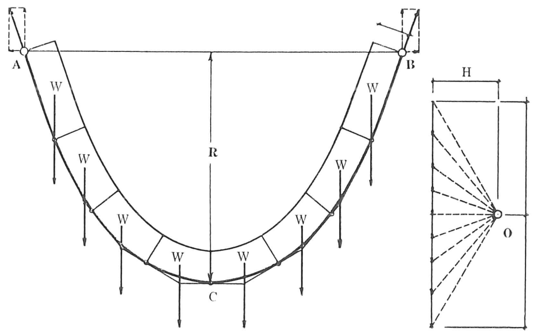
B. That the material of the membrane obeys Hooke's Law. That is: within certain practical limits, strains are proportional to unit stresses, or deformations are proportional to the forces.



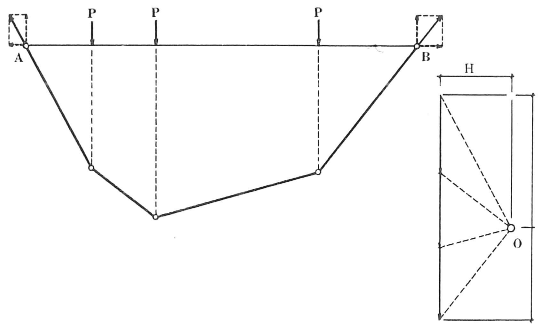
1. Flexible thread.



2. Parabola of equilibrium. Uniform load along the chord.



3. Catenary of equilibrium. Uniform load along the thread.



4. Funicular polygon of equilibrium. Concentrated loads.

C. That the membrane is an "isotropic body" and consequently, has the same elastic properties in any direction.

FORMS OF EQUILIBRIUM UNDER LOADS. Assume a given distribution of forces in space, equally distributed and acting outwardly in all directions, similar to hydrostatic pressure. It is possible to equilibrate this system with a system of stresses lying in a certain surface in space that is the form of the membrane.

This interior uniform pressure in all directions will generate, in a spherical membrane, a balloon for example, a stress distribution in equilibrium with the acting pressure, provided the membrane material is of sufficient strength.

The same system of external forces can also be equilibrated with a membrane of cubic shape or many other closed forms. In any such membranes a definite and unique system of internal stresses will exist in equilibrium. Each will have a particular deformation compatible with the general equation of membranes. In each case different configurations of the internal membrane stresses will be in equilibrium with the same system of external forces.[3]

In the case of membranes it is also possible to make the same assumptions concerning "frozen" forms. That is: A membrane deformed under certain system of loads can be "frozen" without altering the internal stresses previously determined. Analogously the external forces and consequently the internal stresses can be reversed.

From the preceding considerations—the noted analogy notwithstanding—a fundamental difference between the behavior of threads and the behavior of membranes—between systems in a plane and systems in the space—is evident.

Threads: For a given system of external forces, there is only one possible equilibrium configuration resulting in pure tension or pure compression.

[3] It must be pointed out, however, that only one of these different equilibrium configurations will be the most suitable for a given system of external forces. In the example mentioned, this is, of course, the sphere, because this is the only shape for the described load conditions in which the internal stresses will be the same in any point of the surface.

Membranes: For a given system of external forces, there are an infinite number of equilibrium configurations resulting in internal stresses of pure tension or pure compression, but only one will be the most efficient.

CONCLUSIONS:

A. Under the conditions already described, in a particular flexible membrane, for a particular load situation, there is only one configuration of equilibrium. All the internal stresses are tensile stresses.

B. For the same load system acting on N different flexible membranes, there are N different configurations of equilibrium, but still the internal stresses will be tensile in any point of the membrane and in two perpendicuar directions.

C. For a particular "frozen" membrane, there is only one load system that produces the internal tensile stresses identical to those in the former flexible membrane. All the proportional load systems affected by *K* shall produce proportional internal stresses.

D. If a load system changes its sign, the internal stresses of the frozen membrane will be the same but with reversed sign (pure compression).

E. A particular "frozen" membrane can have infinite different systems of internal stresses (tensile, compressive or both) correlative to infinite systems of external forces and not only one as in the case of the "frozen" thread.[4]

II A TYPE OF MEMBRANAL SURFACE

The membrane is the principal element of the structures discussed in this article. The main characteristic of the membranes in question is their shape in the sense that such shape is not a pre-design selection, but rather the result of assumed required conditions of construction and structural efficiency. In other words: a given geometrical surface has not

[4] It should be remembered that membranes have the characteristic of being surfaces of which the thickness is very small in relation to the other dimensions. Therefore, they are not suitable to absorb concentrated loads.

been adopted *a priori* and then transformed into a structure, but rather, constructional and structural questions were formulated initially and then a form developed to satisfy the problem.

The result is a surface of double curvature, different, indeed, from forms of this nature conceived as pure geometrical abstractions.[5]

To simplify matters without altering the basic concepts, only a typical membrane surface with a circular edge and specific conditions for vertices is described here; however, this particular case has been selected because structurally, the circular perimeter is the most efficient. But variations of the same principle are infinite as is briefly shown further on in the discussion of frozen membranal forms.
The typical membranal surface is generated in the following manner:

A. A membrane of a pre-established shape is supported on its edge by a rigid circular ring contained in a horizontal plane.

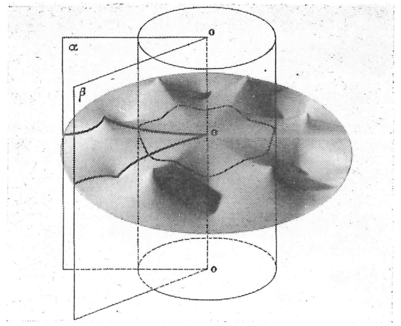
B. By means of upward and downward vertical forces applied on small areas and on convenient points, the membrane is tightened, taking its final shape.

The double curvature skin obtained can be defined a follows: (fig. 5)

Assume a surface that is circular in horizontal projection. This surface has low and summit vertices, under and above the horizontal plane determined by the circular ring. Low vertices are equidistant from center. Summits are equidistant from center. The distances in both cases are approximately between ¾ and ⅔ of the radius. Low and summit vertices are alternated around the center; therefore, they are equal in number. They are also equally spaced.

A vertical radial section through a low vertex determines a directrix composed of two downwardly concave lines: one from the center to the vertex, the other from the vertex to the perimeter.

[5] There are an infinite number of different double curvature surfaces. Some are very well known. All can be defined by one or more mathematical functions $F(X, Y, Z)=0$, even before intuition can conceive them as forms. All these forms are abstractions, their introduction into the field of structures requiring simultaneous consideration of their position in relation to the loads distribution, dimensions, and conditions of supports.



5. Vertical and cylindrical sections on the surface.

A vertical radial section through a summit vertex determines a directrix composed of two downwardly convex lines: one from the center to the summit, the other from the summit to the perimeter.

Tangents to these curves at the center are all horizontal; curves can be of any nature.

The whole surface can be considered divided by the radial directrices in n equal sectors, n being the sum of all vertices. Two sides of a sector are consecutive directrices (low vertices: concave lines; summit vertices: convex lines). The other side is a 1/n segment of the circumference.

To define the surface of a sector, cylindrical sections are taken (axis of cylinder is a vertical through the center of the membrane).

A cylindrical section is a concave-convex curved line. Taking successive cylindrical sections from the center outwards, it is found that up to a certain limit these lines are on a horizontal plane. Concavity-convexity are increasing toward the vertices where they reach the maximum. From there, they decrease gradually until they become contained in the horizontal edge. Any tangent of the section on the radial directrices is a horizontal.

III TENSILE MEMBRANAL STRUCTURES

DESCRIPTION AND BASIC PRINCIPLES

The structure is essentially a pre-stressed membrane of double curvature, constituted by the following elements:

Membrane
Rigid outer ring
Caps
Supporting poles
Foundations, anchorage, drainage

MEMBRANE. In canvas, sheet metal or a net of wire or cables. In the latter case, a non-structural skin should be added. The membrane is in tension, caused by its own weight and by downward and upward vertical forces transmitted by the supports. Its circular edge is anchored to a rigid outer ring.

RIGID OUTER RING. In metal tubes or lattice work. The ring represents the perimetral solution of continuity of the flexible membrane.

CAPS. In metal. They are members that connect the membrane with the supports. Their functions is to distribute the stresses around a given area of the membrane, otherwise membranal stresses will be infinite at the point of connection.

SUPPORTING POLES. In metal or in wood. Tubular or lattice work. They are in compression. Disposed in V shape. Ends are hinged.

FOUNDATIONS, ANCHORAGE, DRAINAGE. In concrete and steel. Transitory structures can be anchored to the ground using ordinary tent methods. It should contain the drainage; that, in the case of non-permanent shelters, can be simplified to a canvas sleeve.

The special characteristics of the structure are:

A. *The membrane* is stretched upward and downward. This provides a great rigidity and minimizes the flapping or vibrating effect caused by wind.

B. *The rigid outer ring* absorbs the peripheral membranal tensions.

C. *The supporting poles* are connected to each other, forming a closed system, thus eliminating torsion on the horizontal plane.

D. *The foundations* absorb only vertical loads when two adjacent poles are in a vertical plane. It is interesting to observe also that it is precisely on this point where the membrane is anchored, thus, upwards vertical forces are coincident with the vertical resultant of the downwards inclined forces. Anchorage and supports are unified in one element simplifying foundations.

E. *The drainage* has been provided within the structural shape. The total surface naturally slopes toward the lower points of anchorage, thus preventing the accumulation of water on any area of the skin.

F. *The whole system is symmetric* with an axis of symmetry running through summits of center, low vertices and center, thus assuring stability and structural efficiency.

Although two typical tensile examples are illustrated here, great varieties of structures can be obtained within the established limitations and within requirements of symmetry which is desired for an even distribution of stresses in supports and ring.

Considering given conditions: span, load distribution, materials, soil, the variable elements of design are:

VERTICES. Low and summit vertices can vary in number, positioned in different points and having different heights. The number of low and summit vertices in each case is always the same (i.e.: 4 and 4; 5 and 5; 6 and 6, etc.) Position must be considered in relation with the center and exterior ring.

Vertical distance between low and summit vertices is a significant factor in the membrane stresses, because its variation changes the curvature in any point of the membrane.

MEMBRANE. There are two kinds of variations in the form of membrane, one that is dependent on the vertices (number, position, height), and the other inherent in the membrane itself. Without moving vertices, the central point of the membrane as well as the exterior ring can be raised and lowered as desired within certain limits. Also, as the final form of the stretched membrane is a result of a pre-established shape, different curved surfaces can be obtained accordingly.

The form of these membranes cannot be generated by deformation of a flat plane under the action of concentrated forces. Experiments in circular membranes fixed to a rigid horizontal ring, and with a load applied in a small central area, show radial wrinkles between the point of application of the load and the ring. These wrinkles are caused by internal compressive stresses. Besides, in these experiments, materials like rubber were excluded because of their great deformability. Therefore, any concentrated force acting on the membrane produces remarkable local deformations. The shapes obtained in this manner have no further structural interest.

The problem is to find a shape for the membrane which, when stretched, would take a required form and proportions. Models were built at the scale of 1/20 and 1/40 of the full size, in order to study this problem as well as the general behavior of the structure.

Three ways have been found to originate the required curved surfaces:

A. Making the total surface of smaller flat pieces of material assembled together. This process involves cutting and joining, and can be applied to all materials whether or not they absorb bending moments.

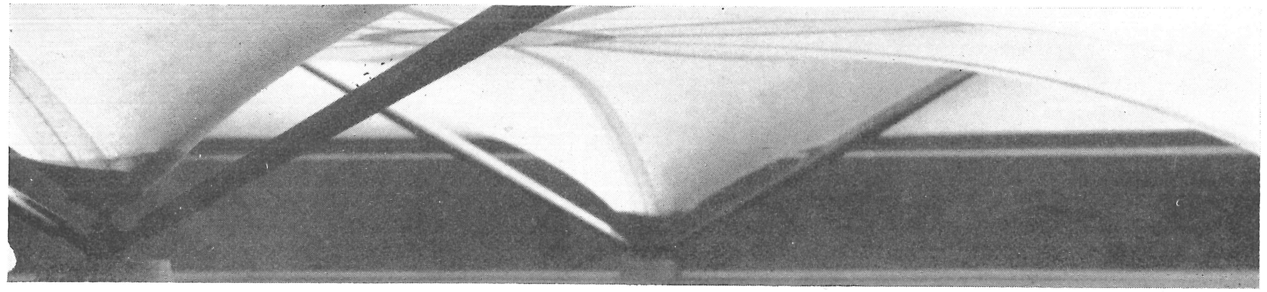
B. Making the total surface of smaller moulded pieces of material assembled together. This process involves cutting, moulding and joining and is suitable only for materials capable of absorbing bending moments.

C. Generating the form with linear elements (cables, rods, wires) similar to a net. In this case a non-structural skin should be added because the membrane is reticular. Or, a solid surface should be built up on the net in the case of solid membranes, such as thin shells in reinforced concrete.

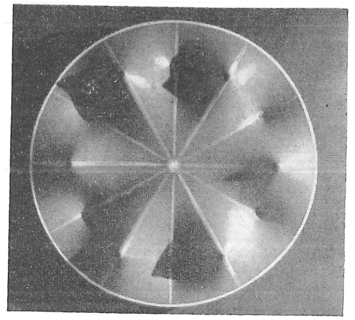
In the initial studies, the first method indicated above was adopted for practical reasons; that is, to divide the whole area in such a manner as to permit the development with flat pieces assembled together. It should be observed that this division depends not only on the geometry of the form, but also on the characteristics of the material used. There are innumerable layouts for the same shape and material.

On the experimental models, the total area is divided in equal sectors defined by a pair of low or summit vertex radii. The number of sectors is equal to the number of vertices of the same nature; for example: for six low vertices, there are six sectors, etc. These sectors can be made of a single flat piece (as is done in the model) or of parallel strips of material.

After many attempts, a method of division has been developed which permits one to control with great precision the final shape and proportions of any membrane previously designed. Methods of erection of a structure should be considered in relation to the overall dimensions, materials, available equipment, etc. In the present case, it is in course of study, a project for a relatively small structure: 100 feet diameter, 70 feet span, membrane in army duck. Its method of erection is not discussed here.



6. Tensile structure: interior view.

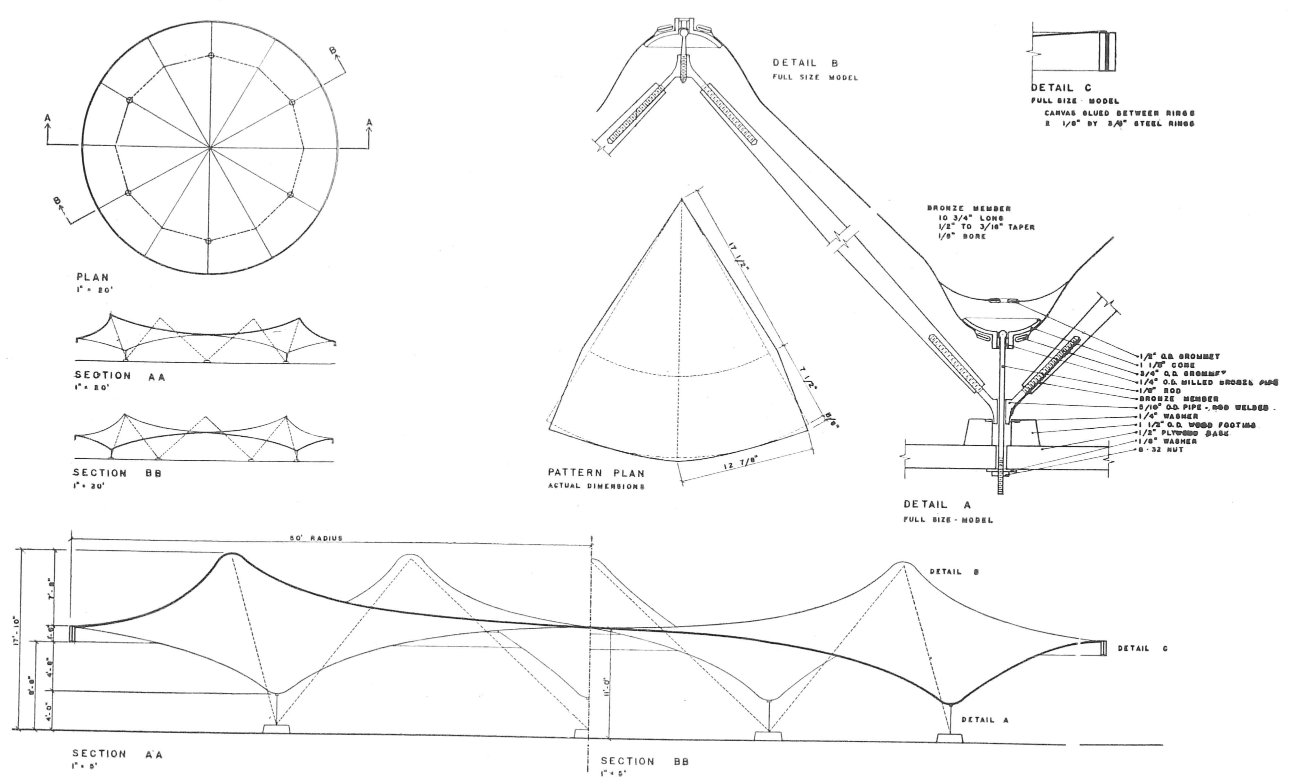


7. Plan.

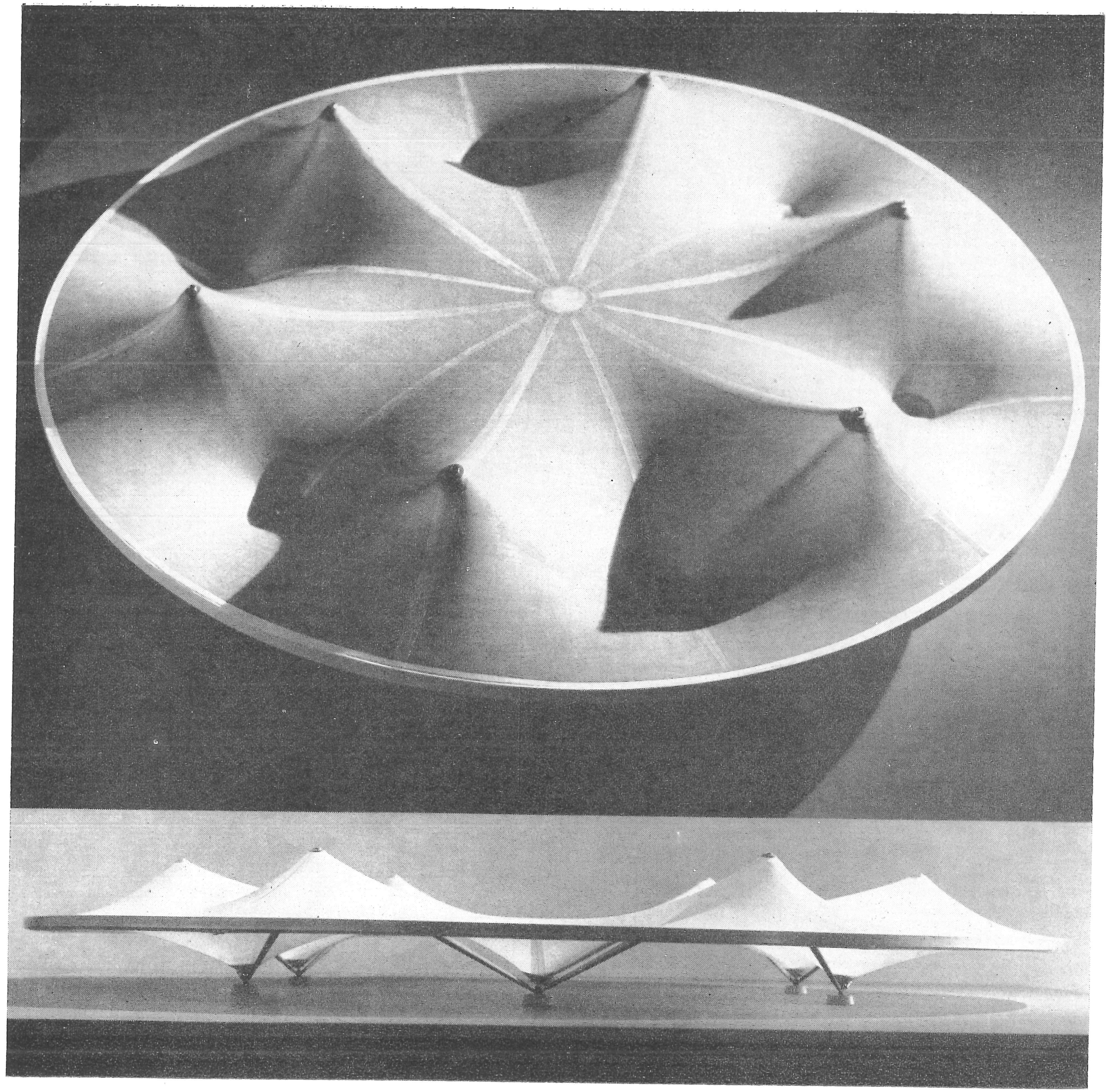
TENSILE STRUCTURE I

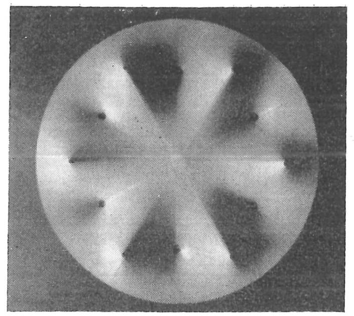
is illustrated in drawings and photographs of a model. (fig. 7 to 10) Membrane is in canvas, although as has been stated, sheet metal can be used. Diameter=100 feet; six low vertices and six summits. A flat sector of the membrane is shown. For the model, the sector was made of one piece of material. For a final membrane, strips of army duck, 26 inches on center are projected (Width: 28½ inches. Weight. 12 ounces per square yard. Count: 46½ x 35. ply: 3 x 3. Breaking strength: 1 x 1 x 3 grab method. Warp: 281 lbs. Filling: 245 lbs.).

9-10. View and elevation of the same structure.



8. Plan, sections, details. Drawings are based on the model.



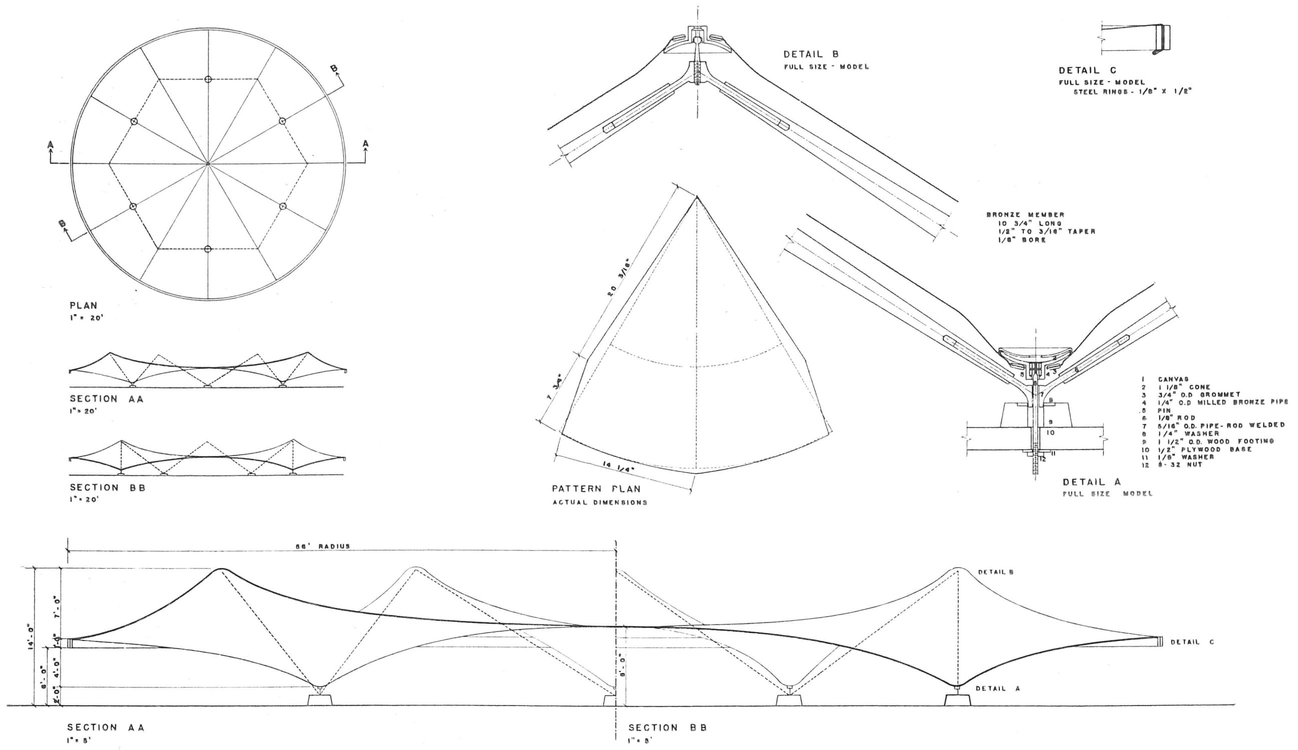


11. Plan.

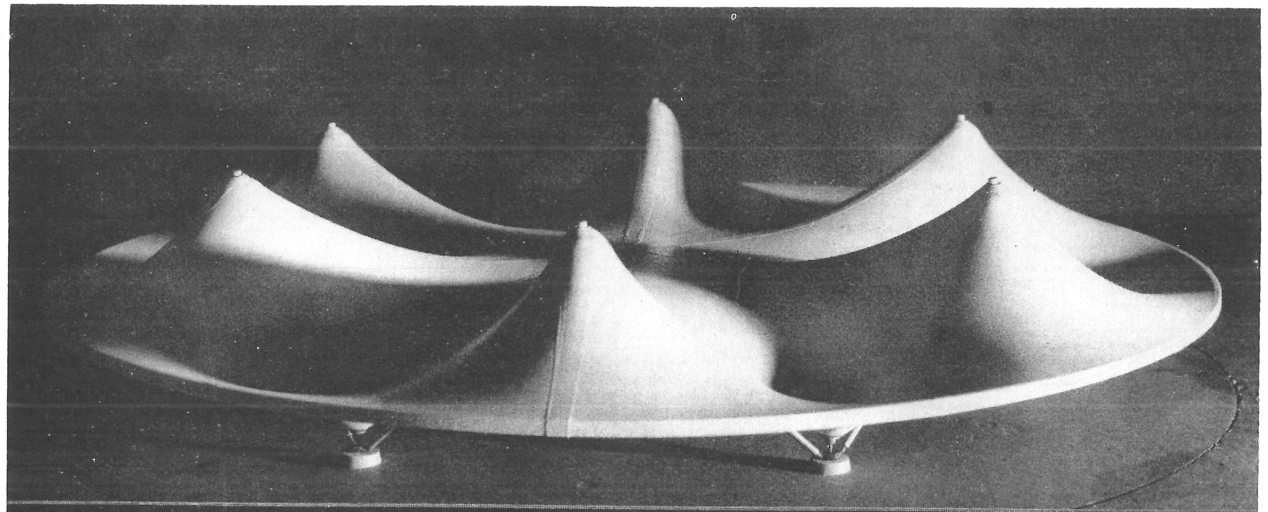
TENSILE STRUCTURE II

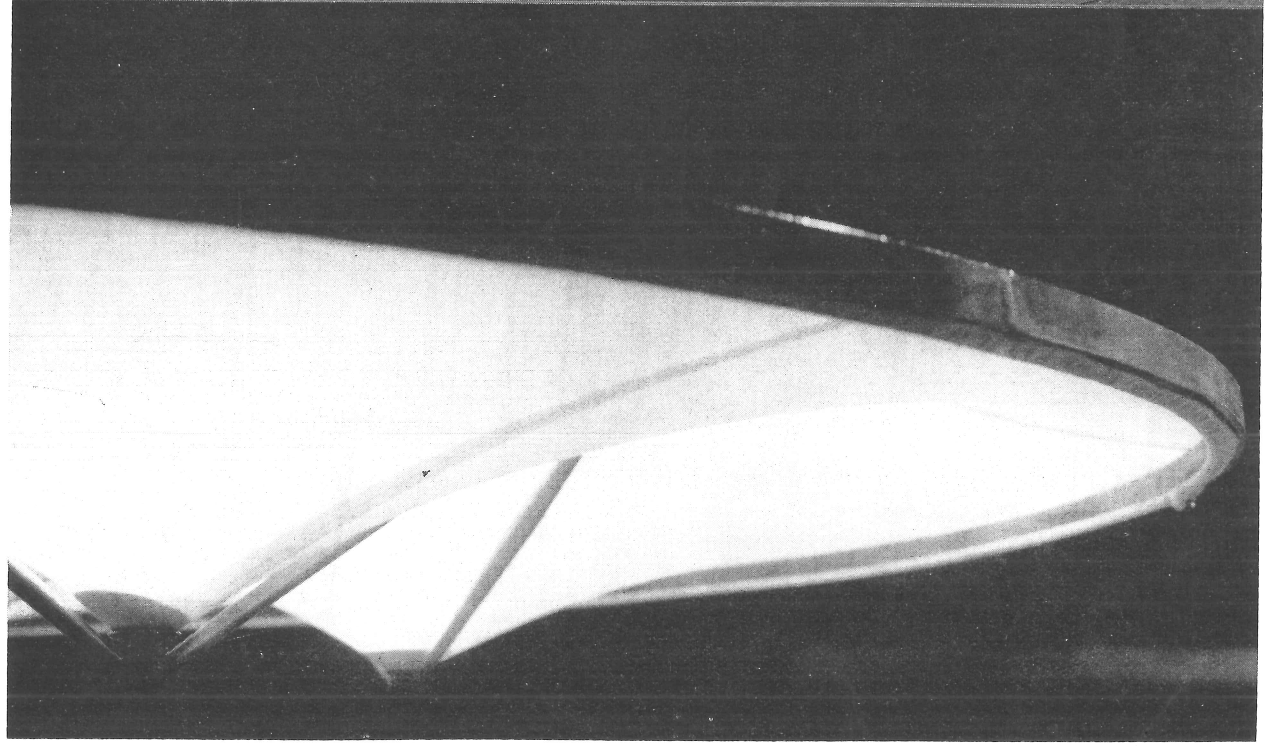
Membrane in canvas. (fig. 11-12) Diameter: 112 feet. Similar to Structure I. However, there is a difference in positioning the supporting poles. Each pair of poles concurrent to a low vertex is contained in a vertical plane and, consequently, the resultant is vertical to the ground. In the case of the precedent structure, this does not occur and, therefore, the foundations must absorb horizontal forces.

(Opposite page) 13. Top: A six-support structure with higher summits. Model is in canvas. **14. Bottom:** Interior view.



12. Plan, sections, details. Drawings are made after the model.





RETICULAR STRUCTURES

There are no substantial differences between systems of membranes of continuous surface and reticular systems (network or lattice work). In the first group, there is an internal distribution of stresses that cannot be determined *a priori*. In the second group, the geometrical disposition of linear elements represents *a priori* lines that the internal stresses shall follow.

But despite the possible difference in the distribution of internal stresses, both groups have in common the condition that such stresses are contained in a surface and not in a volume.

Two types of reticular structures can be considered:

A. *Networks* constituted of flexible members, capable of absorbing only tension (wires, cables or rods).

B. *Latticework* constituted of rigid members capable of absorbing either tension or compression.

In both cases, a water-proof, non-structural skin should be added to complete the cover. If this skin becomes structural, the membrane is transformed into a continuous surface reinforced with a grid of ribs.

These two types can be considered mainly for large span structures where the dead load is an important factor of design and which can be reduced if a relatively light non-structural cover is supported by a net of linear elements. The utilization of permanent materials, like sheet metal, seems appropriate for the cover.

The layout of the net presents problems that are rather difficult to analyze separately and on paper. For preliminary attempts there are no other means than experiments carried on with small models, whose scale can be increased gradually. Models have been built from 24 inches to 10 feet in diameter. The layout of the net should satisfy primarily established requirements of construction, such as:

A. *Generation of the given form.*

B. *Feasibility and simplicity of construction.*

C. *Rational utilization of different materials:* steel wires, cables, steel rods, which, although they can be used structurally for the same function, demand a specific technique of construction in each case.

D. *Control of the density of the net.*

The first and last conditions can be studied on a relatively small scale. The others require at least the construction of full-size portions of the structure. The study of the net is also related to structures in reinforced concrete that are discussed in other sections.

The few examples illustrated here show merely a tentative approach. Most are preliminary studies and therefore they do not include the rigid ring. The solution of continuity of the membrane is provided by flexible members anchored to the ground in different ways.

On structures of large dimensions where the rigid ring is adopted, it could rest directly on the ground or have additional supports.

Finally, it must be pointed out that the possibilities of reticular structures are singularly interesting because it seems that the advantages of tensile structures over other systems become remarkable in long span constructions, at least with reference to particular aspects of the problem that cannot be discussed here.

Beams, trusses, arches, in this progression, have successive practical limits of span; tensile reticular structures may surpass these limits, as in the case of suspension bridges which have a greater scale of span than any other system.

15-16. Preliminary study without rigid ring. Hexagonal plan. Six supports and six anchorages on the ground. A reticular version of the skin structure of page 23.

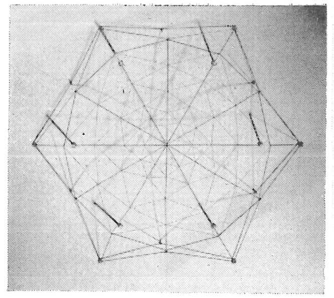
17. Study for a circular plan with central opening similar to structure on page 23.

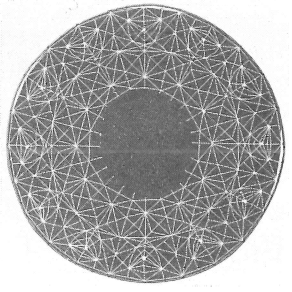
18. Layout of the net with rigid ring, six summits, six supports.

19. Detail of a flat sector of the preceding structure.

20-21. Another preliminary study without ring. Twelve supports, twelve anchorages on the ground.

22-23. Preliminary schemes without ring, showing different layouts of the net.





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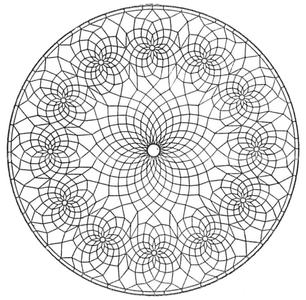

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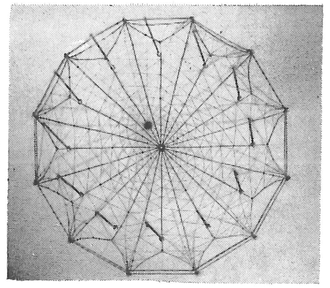

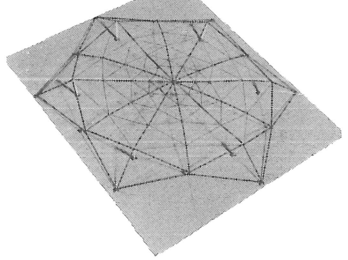

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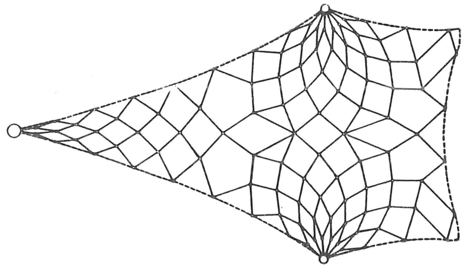

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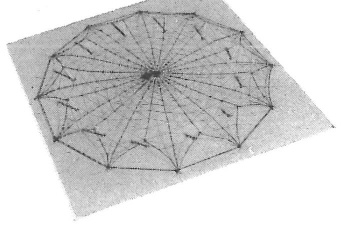

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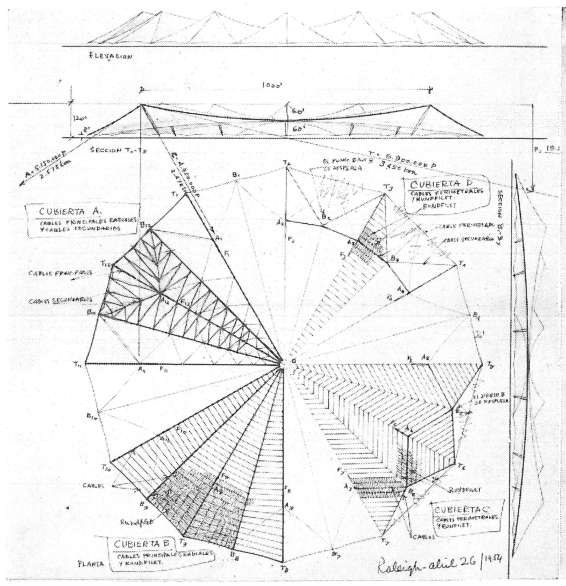

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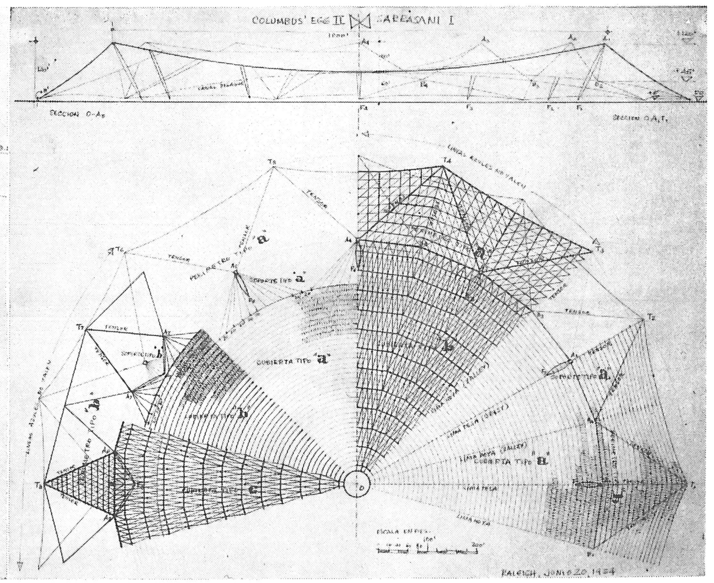

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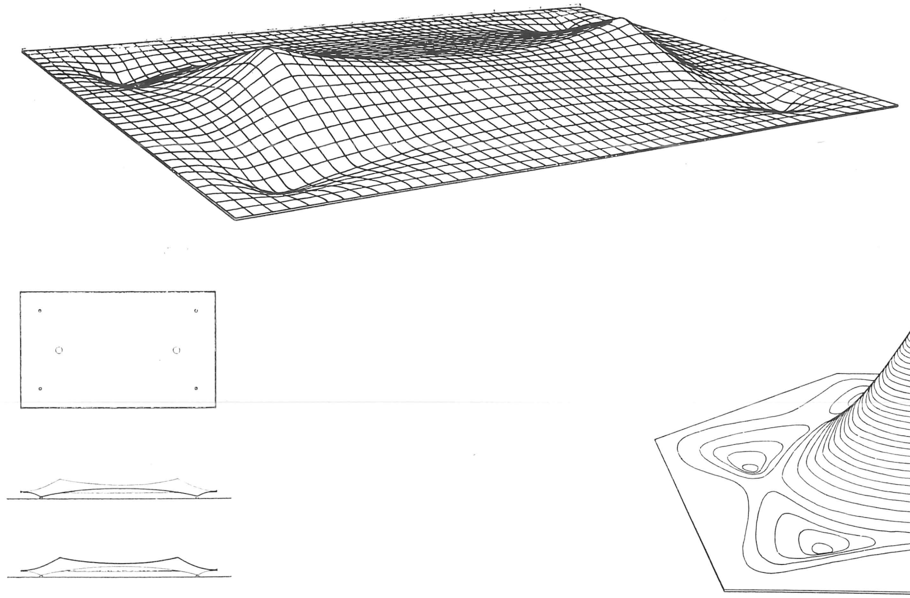
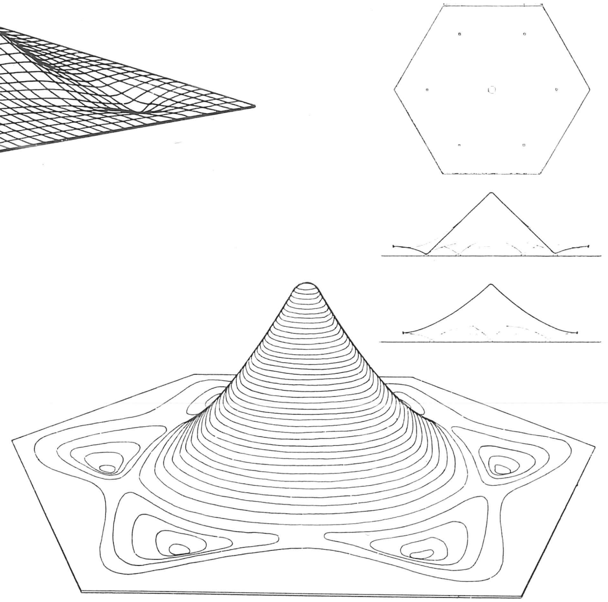


24. Rectangular perimeter. Two summits. Four low vertices.



25. Hexagonal perimeter. One summit. Six low vertices.

IV RIGID MEMBRANAL STRUCTURES

FROZEN MEMBRANAL FORMS GENERATED BY UPWARDS AND DOWNWARDS FORCES

In accordance with the general principles of equilibrium discussed in the preceding sections, it seems possible to originate frozen membranal forms under the following specific conditions:

A. *A membrane of a given shape is supported by a rigid frame contained in a horizontal plane.*

B. *Upwards and downwards vertical forces are applied in small areas and on convenient points. The membrane is tightened, taking a form that is in equilibrium.*

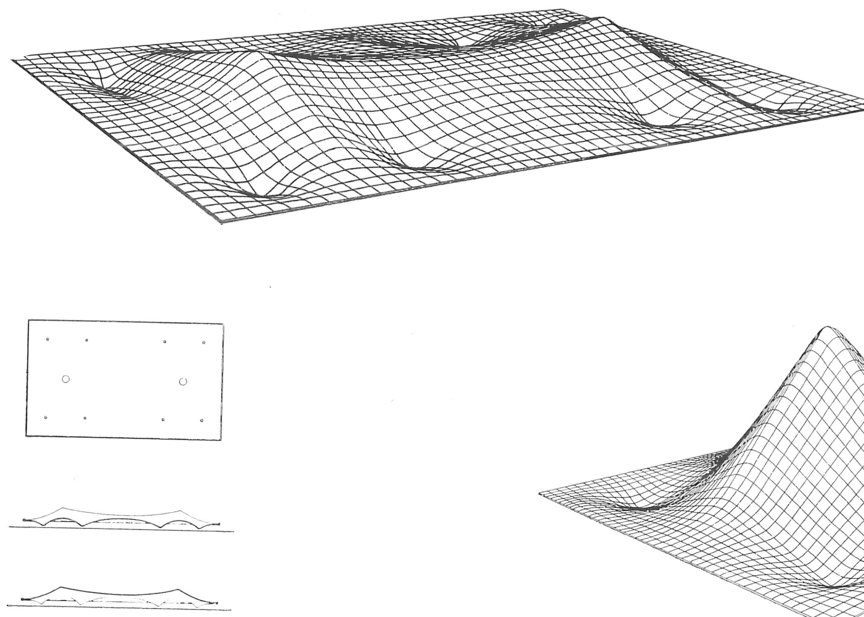
C. *The membrane is solidified and the forces released.*

D. *It can be assumed that the shell obtained is capable of absorbing tensile as well as compressive and shearing stresses correlative to an infinite number of systems of external forces.*

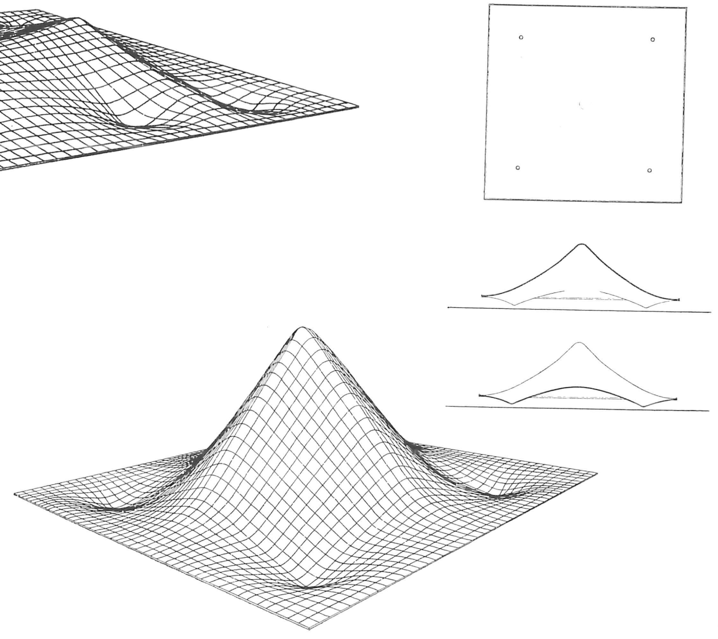
Several experiments were made on this basis. Some examples are shown in photographs and drawings.

It is necessary to point out that they are merely equilibrium forms at this stage. Only a further study will determine which of these shapes has the possibility of being developed as a structure. The main purpose of the experiments was to find a way to generate structural shells in reinforced concrete that could be built without, or at least with a minimum of, form work. In this case, the membrane would be made of a wire mesh supported on the frame and vertices. Prestressing would be achieved by vertical displacement of the vertices or by other means. Concrete would then be spread in layers with a gun until the shell was built up. When the concrete, or rather, the mortar, had settled, forces would be removed. In the next section this proposition is discussed in more detail.

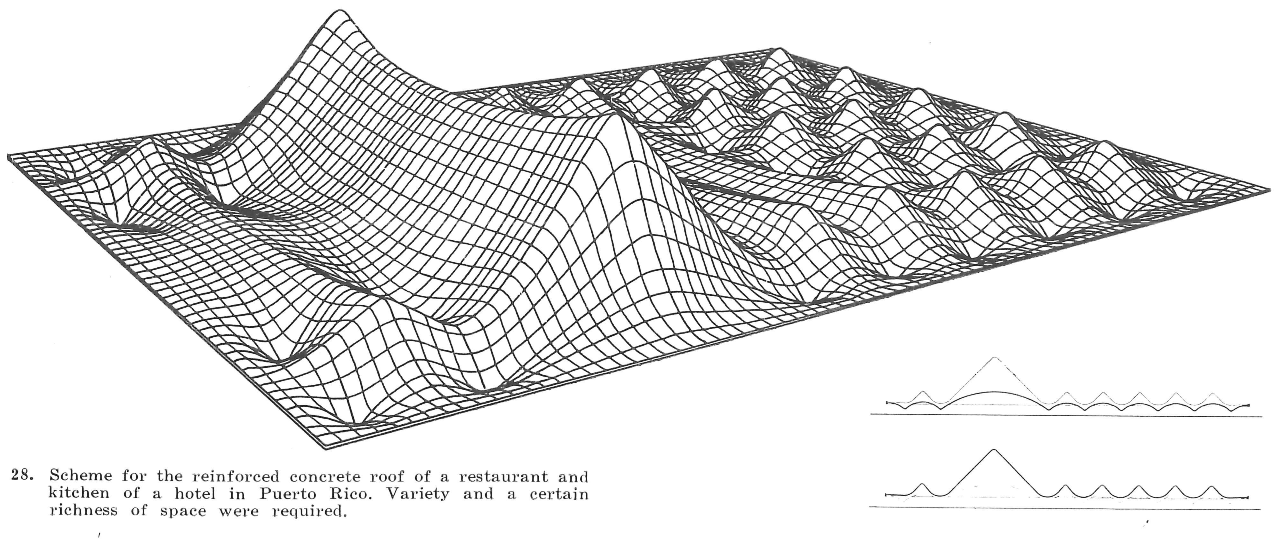
Other characteristics of this form is that low vertices are natural points of support on a horizontal plane:



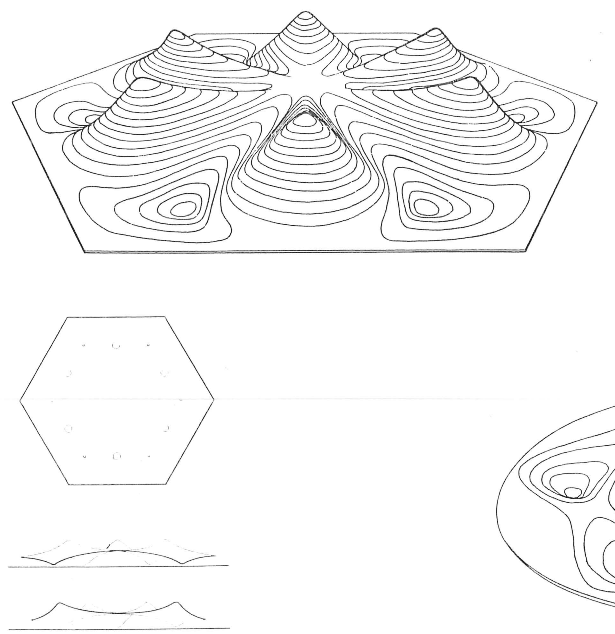
26. Rectangular perimeter. Two summits. Eight low vertices.



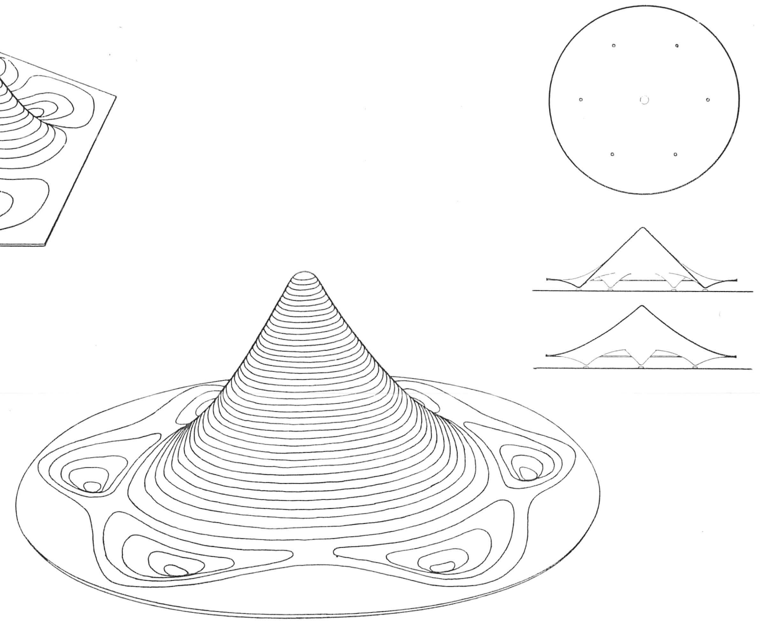
27. Square perimeter. One summit. Four low vertices.



28. Scheme for the reinforced concrete roof of a restaurant and kitchen of a hotel in Puerto Rico. Variety and a certain richness of space were required.



29. The same form of the tensile structures shown before is adapted here to a hexagonal perimeter.



30. Circular perimeter with a central summit and six low vertices.

supports and shell are integrated into a whole. Because of their position and conical configuration, they are highly efficient. A comparison can be made with the usual thin shell structure, where the shell rests on members (columns) that are on the edge of the shell and foreign to the system itself—a solution that has not been improved upon but is nevertheless statically known.

It can be observed in the few examples illustrated that variations are infinite. Although forms of different nature can be obtained, the search was limited only to surfaces of double curvature that present obvious advantages: ease of construction with this particular technique and ability to absorb internal compressive stresses. Relations of symmetry were naturally adopted in these first attempts because they simplify geometrical problems and apparently lead to better static conditions. However, symmetry is not mandatory.

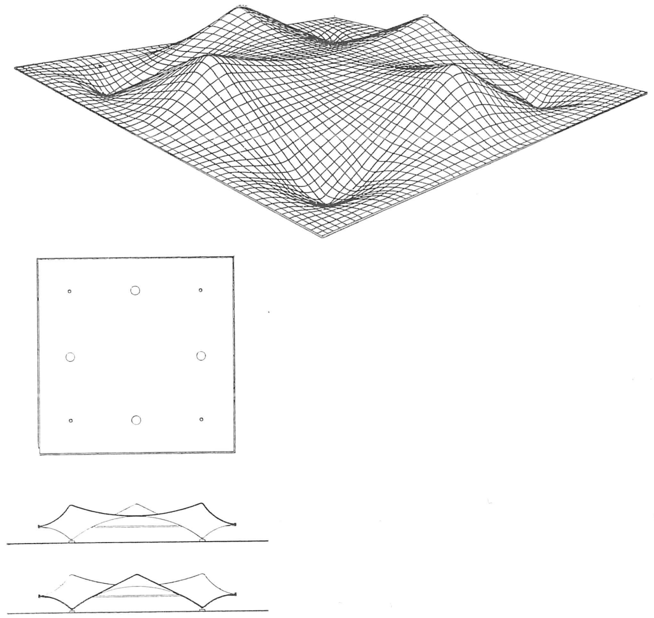
The forms are functions of the combination of the following variables:

A. PERIMETER. The best structural shape for the perimeter is the circle, since it is a form highly capable of absorbing the symmetric edge membrane stresses. This does not eliminate the possibility of deformation (elipsoidal, trifolium, quatrifolium). Nevertheless, assuming that it is practically feasible to achieve regidity with a frame composed of straight lines, (as in the case of some thin shells) any polygon can be considered.

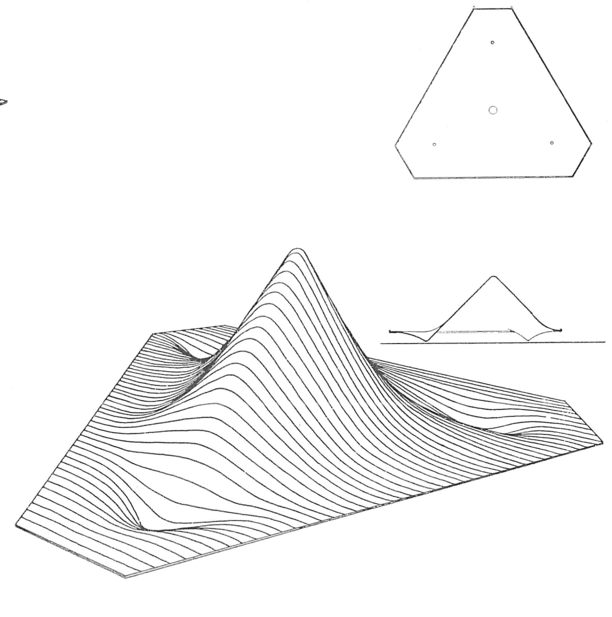
B. VERTICES. Low and summit vertices can vary in number, positioned in different points and of different heights. For reasons explained before, vertices are not sharp, but small spherical surfaces. Considering the vertical distance of the vertices to the horizontal plane determined by the perimeter, it is found: a) that the form becomes flatter when these distances decrease; b) for a great value of the summit vertices and small of the low vertices, the space contained under the skin is much more enclosed than when these relations are reverse.

C. CURVE DIRECTRICES. These are the lines between two related summits or low vertices or between vertices and a particular point on the edge. They are concave and convex. Because the surface is of double curvature, each point belongs to two different curves determined by the intersection of two perpendicular vertical planes. The sum of the rise of the two curves is constant. Thus: concavity and convexity are in inverse ratio.

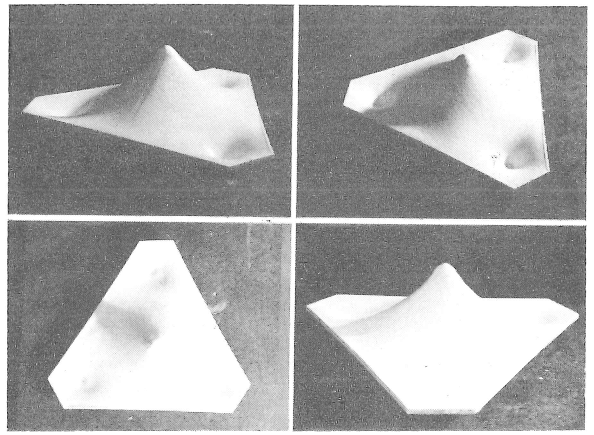
There are endless variations as can be seen in these few examples. Because it is beyond the scope of this article, devices such as perforations, which are a relatively simple operation in a membrane, and which will create sources of natural light, are not included here. Finally, it goes without saying that these units can be also repeated and organized ad libitum.



31. Another variation with square perimeter. Four summits, four low vertices. A reinforced concrete model of this form is illustrated on page 19.



32. Triangular perimeter. One summit, three low vertices.



33. Model of the above triangular perimeter form. Built in canvas with a metal frame. The form was "frozen" by applying a special paint.

MEMBRANES IN REINFORCED CONCRETE

Reinforced concrete is undoubtedly an excellent material for the construction of membranes (thin shells). Its main economical disadvantage lies in the fact that form work is required. In the specific case of application to thin shells, it is known that a costly structure in wood or metal must be erected first to support temporarily a skin of 2 inches—more or less—in thickness.

The typical membranal forms illustrated here have been developed as an attempt to overcome this difficulty. It is plain that the method suggested in this section demands a careful study and only full-scale constructions can give a proper answer. Nevertheless, a previous investigation in models of reinforced concrete was necessary before attempting experiments in larger scale.

Models in reinforced concrete have been built with forms and without forms, not only for the purpose of studying constructional problems but also for loading tests.

A constructional method is outlined as follows:

A. *Foundations are constructed for the low vertices.*

B. *A minimum of scaffolding is erected to support edge and summits.*

C. *A rigid steel frame (edge) is installed.*

D. *Wires are stretched following the main generatrices of the desired shape and tied to the frame. Also, generatrices are connected with secondary lines of wires.*

E. *The whole net is tightened by lifting up the summits or pulling down low vertices.*

F. *A steel mesh is tied to the wire net following the form.*

G. *The whole steel reinforcement is tightened for a second time by the same procedure.*

H. *Concrete is spread in layers, with a gun, until the required thickness of the shell is obtained.*

I. *When the concrete settles, the scaffolding of frame and summits can be removed.*[6]

The process, briefly described, involves, indeed, if not complicated, delicate operations such as the control of tension in the wires, handling and application of the mortar, etc.

A variation of this method contemplates almost the total elimination of scaffolding in this manner: the rigid steel frame (edge) will lie horizontally on the ground. The net is prefabricated in sections with wires, cables, or rods of a pre-established dimension. These sections will be assembled and fixed to the ring. Net and ring will be lifted by means of hydraulic jacks applied on given points of the net (summit vertices). Finally the steps E to I follow.

But, in the last analysis, any specific system of construction will be determined by factors such as materials used for the net, method of weaving, and especially by the overall dimensions of the structure.

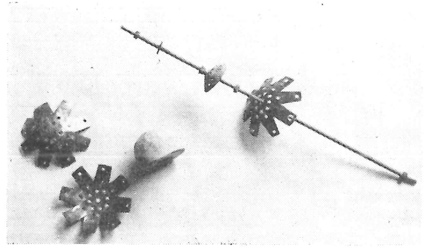
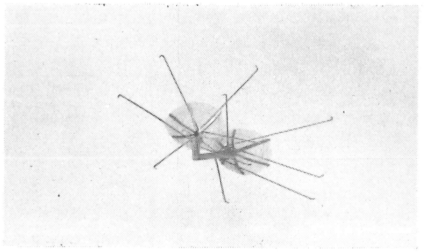
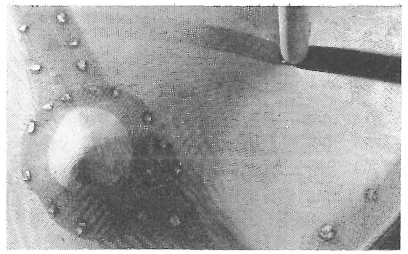
A model built without forms is illustrated. (fig. 41-42). Mortar was applied in layers with a trowel. Main characteristics: square plan. Frame: ⅛ x ¼ inches steel plate. Generatrices: wire. Galvanized wire mesh. Mortar: 1:2, cement, sand. Sand 18 mesh. The shell does not have a uniform thickness: variations approximately from ¼ to ⅛". (Identical to the membrane in fig. 31).

Five models of circular plan were built using plaster forms, because a smooth finished surface was required to apply "stress coats". Plaster forms were cast directly from the canvas model shown in fig. 7-10. This set of models has the same characteristics: Diameter: 50 inches. Thickness of the shell: approximately ⅛". (Dimensions are 1/20 of the actual size of the structure.) Six summits and six low vertices. Mortar was applied with a brush.

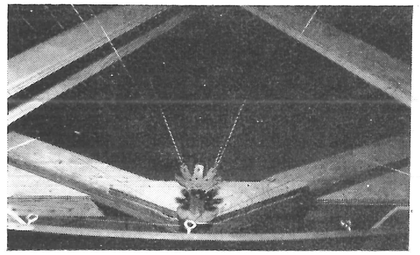
A circular model without forms is in course of construction: Diameter 10 feet. Thickness ¼" (Dimensions are 1/10 of the actual structure). Tinned steel wire cloth 10 x 10 mesh. .020 wire.

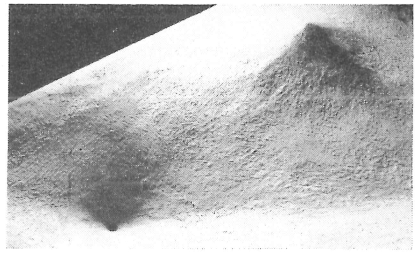
Loading tests of the described models are in progress mainly for a qualitative analysis, at least to determine the isostatic layout. Methods prescribed for the tests: stress coats and strain gauges.

[6] It must be noticed that a particular form will have different structural behavior depending on how summit vertices are supported: by means of the membrane itself; by reinforced ribs contained in the membrane and extended up to low vertices; by independent members, etc., etc.

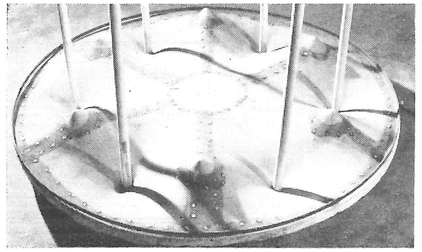
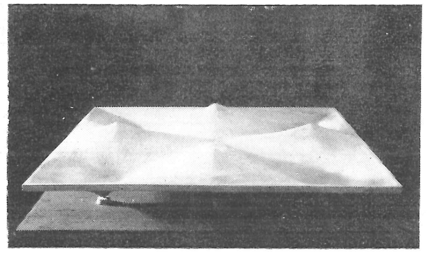
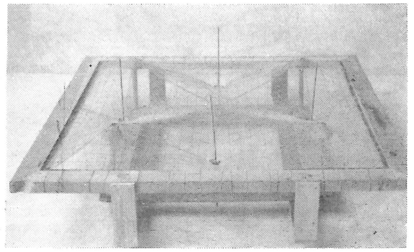




Different aspects and details of the reinforced concrete models:
34-35. Metal caps for the reinforcements of summits and vertices. The rod shown is threaded to move the cap vertically, when stretching the reinforcing mesh. 36. Summits and low vertex of a model built with a plaster form.

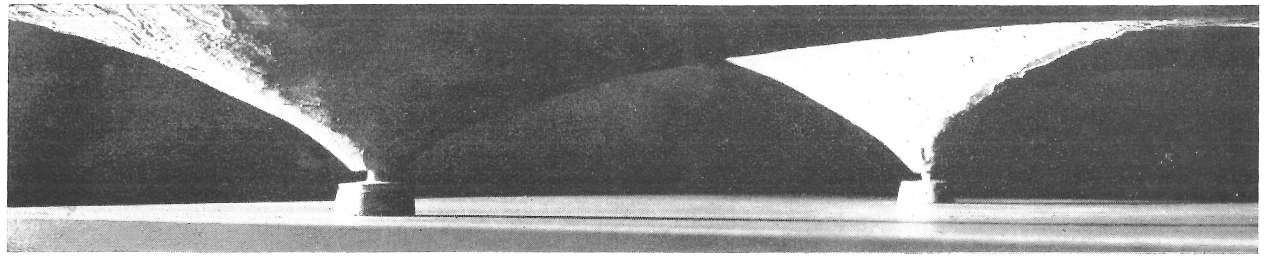




37. Stretching the metal net. 38. Finishing the concrete at summits. 39. Finished summits and low vertex of the square perimeter model, surface was finished with brush.

40. Reinforcing mesh and metal ring on the plaster form, ready for pouring. 41. Square perimeter model built without formwork. 42. The reinforcing mesh of the same model is shown stretched on a wood frame.



43. Interior view of the concrete model showing two low vertices. Surface is rough because it is finished with a trowel.

V MISCELLANEOUS

WIND ACTION

The study of the action of wind upon the structure can be of value due to the "sui generis" characteristics of the membrane: lightness and thinness. Tests were carried out to obtain, in particular, the following information:

A. *Air load distribution over the surface: points of maximum loads, areas of rapid change in the air loads*

B. *Total lift load of the surface*

C. *Over turning moment tendency*

D. *Air flow across the surface*

Reports have not been completed and only a brief note is included on the air load distribution over the surface.

The experiments were run in a single-return wind tunnel, closed circuit, of the aeronautical department of North Carolina State College.

A double skin plexiglass model (25 inches diameter) was used (see fig. 44) with the purpose of registering air pressures on top and bottom of the surface. In view of the symmetry of the structure 52 orifices were distributed in a 30° sector of a circle.

Four orifices in the opposite direction were considered enough to check the readings. Thus, by rotating the model 12 times, for each particular case, the action of the wind in the whole surface was analyzed.

All the tests were run with an 80 mile per hour wind under these conditions:

Test I. With the structure open in the periphery — that is, the case of an open shelter. The action of the wind is simultaneous in both faces of the surface: top and bottom. Direction of the wind is parallel to the summit vertices diameter.

Test II: Similar condition as *Test I* (open periphery). Direction of the wind is parallel to the low vertices diameter.

Test III: With the structure closed on the periphery —that is, the case of an enclosed space. Therefore the winds act only on the top surface. Direction of the wind is parallel to the summits vertices diameter.

Test IV: Similar situation to *Test III*. Direction of the wind is parallel to the low vertex diameter.

Due to the symmetrical conditions of the form, it was enough to experiment with only two directions of the wind:—one parallel to the summit vertices; the other parallel to the low vertices—to obtain any other direction with 30° of difference.

Air load distribution over the surface is shown in the accompanying schemes. (fig. 45-47). Air loads are expressed in pounds per square foot. Forces are perpendicular to the surface. Contour lines follow lines of the same load value. Solid lines are negative pressures (downward forces); dotted lines are positive pressures (upward forces).

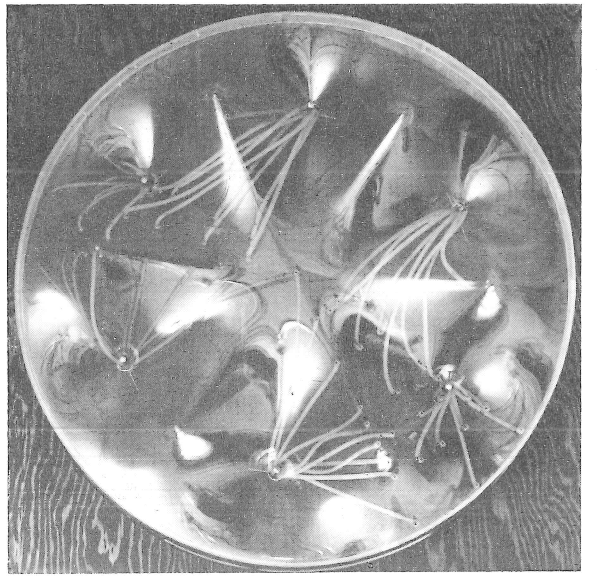
As was expected, the 80 miles per hour wind originates mainly a lifting action on the structure and only small areas of the membrane are under downward pressure. The downward pressure areas are larger when the structure is open at the bottom.

Pressures are not remarkable. Greater values (points of maximum loads) are always on the summit vertices and proximities (up to 40 pounds per square foot in the open structure). Areas of rapid change of the air loads (that increase the internal tensile stresses of the membrane) are also consistently around summit vertices.

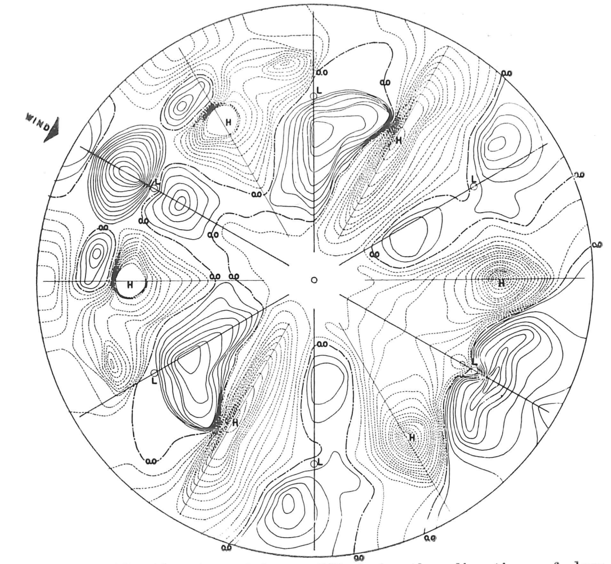
In the proposed structures, summit vertices are points of support and consequently, points of concentration of internal stresses where the membrane requires reinforcements or any other special treatment. Therefore, the location of these pressures is a favorable structural condition.

Small values of pressure are found on the low vertices, central area, and peripheral area of the skin.

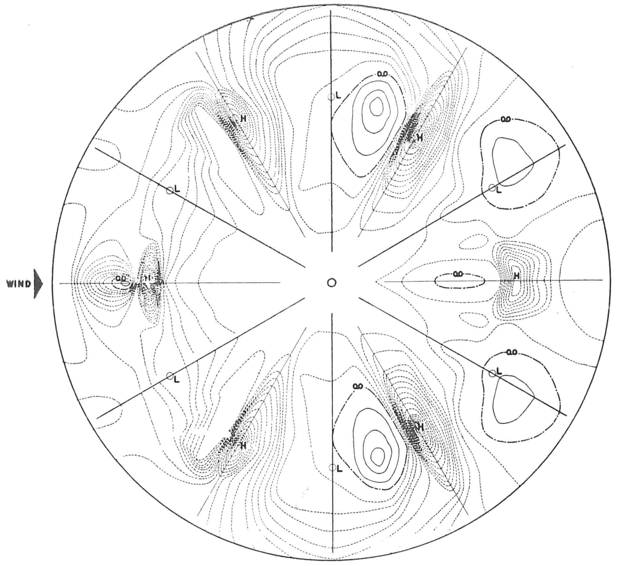
In the open structure: center, 3, average; periphery, 5, average. In the closed structure: center, 2, average; periphery, 4 average. Since the critical zones of the membrane are in the center and periphery, this is obviously again a structural advantage.



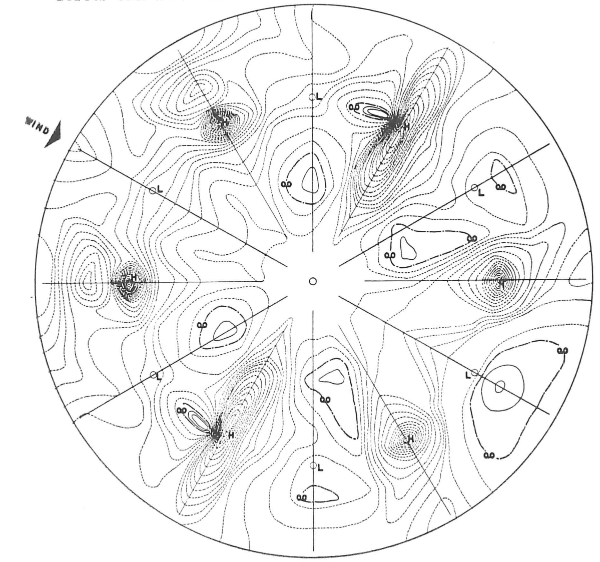
44. Plexiglass model of double skin for the wind tunnel experiments.



45. Test II. Closed periphery. Wind in the direction of low vertices axis. Wind acts only on the top surface. Contour lines follow lines of the same load value. Null pressures are indicated with heavy lines. Six areas of downward forces can be seen.



46. Test III. Open periphery. Wind in the direction of high vertices axes. Wind acts on both bottom and top surfaces. Five areas of downwards forces.



47. Test IV. Open periphery. Wind in the direction of low vertices axes. Nine areas of downward forces. As in all the tests, areas of higher upward (lifting) forces are shown around summits.

CONSTRUCTION OF PLEXIGLASS MODELS

For the wind tunnel tests, a model was built of a double skin of plexiglass for the purpose of registering wind pressures on both sides of the surface simultaneously.

Two methods were tried in molding the plexiglass: A. *With upward and downward pressures in points.*

The jig shown (fig. 48) was used. The sheet of plexiglass was adjusted between two perforated plywood boards and heated. The jig allowed simultaneous pressure to be applied upward and downward to the flat sheet on points correspondent with the vertices. A form was obtained (fig. 49) that, although similar to the typical surface, does not reproduce the desired curvatures. The heated plexiglass behaves like rubber, has great plasticity and, therefore, the forces applied only produced local deformations (cones) around the points of application. This experiment confirms in a rudimentary manner that the membranal form cannot be originated with concentrated loads acting on a flat surface nor from deformable materials.

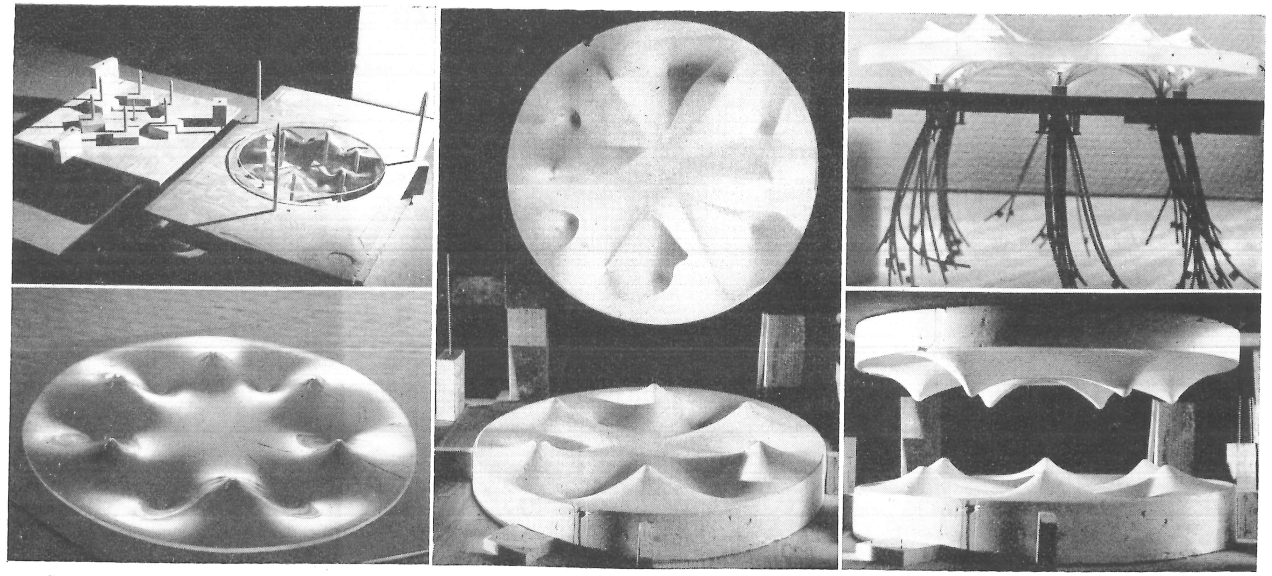
B. *With molds.* A first mold was directly cast in plaster from one canvas model built for the purpose, 25 inches in diameter. The canvas membrane, made of cotton, was rigid enough to support the uniform load of the plaster without deformation.

The male and female forms were assembled (fig. 52), in such a way that one fell vertically into the other by means of a simple and rapid operation. The plexiglass was supported between two plywood sheets and heated, then was introduced on the mold and compressed. The required shape was obtained.

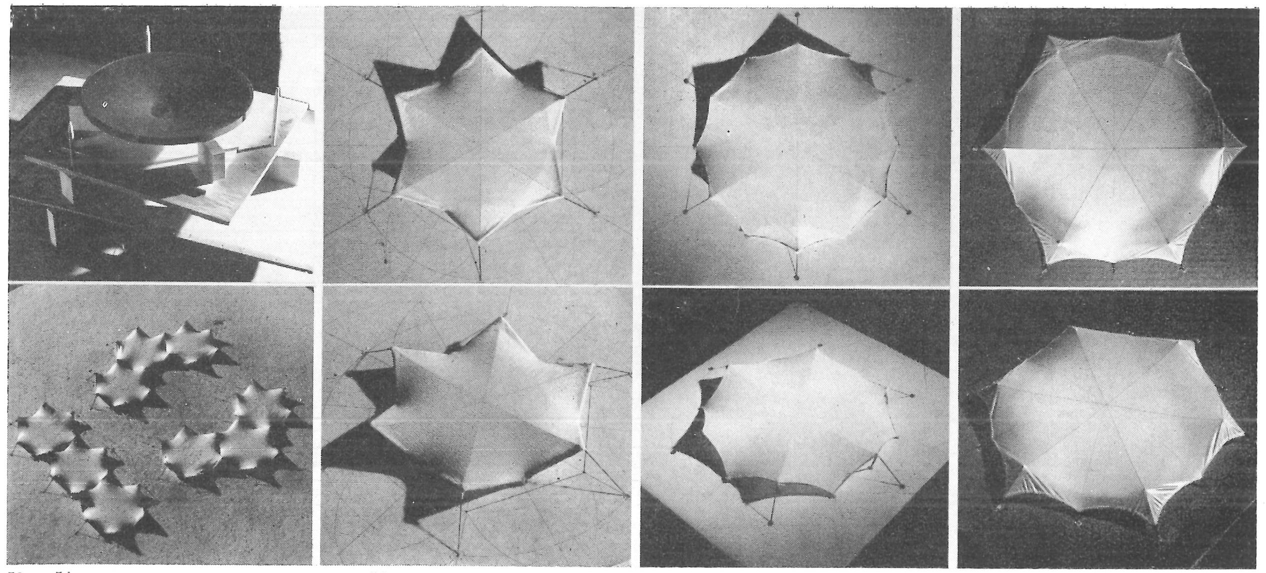
PREVIOUS STUDIES

Many attempts were made before the typical structure was developed. Although carried out in an elementary manner, these studies were always based on experiments and theoretical analysis.

Some of these preliminary attempts are illustrated here since they have an interest in themselves.



Construction of plexiglass models. **48.** Jig to apply simultaneously downward and upward concentrated forces on the flat plexiglass sheet. **49.** Plexiglass form obtained. Photograph shows that deformations occurred only around vertices. Center of form is flat. **50-52.** Male and female plaster forms to apply pressure in the whole surface of the sheet. **51.** View of the double skin model.



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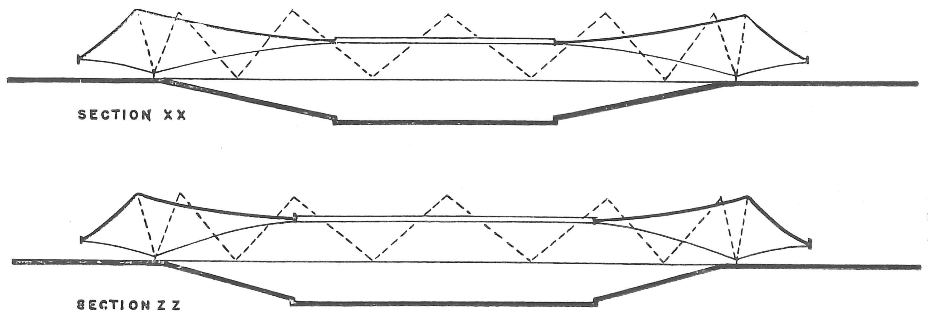

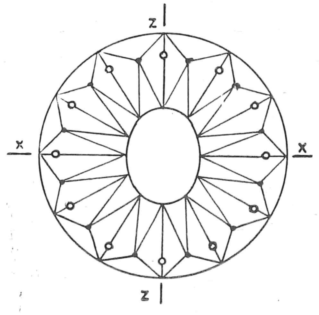

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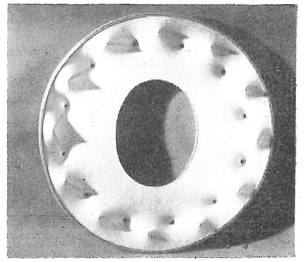
Previous studies. **53.** Stretching the canvas on a rigid ring by means of a concentrated force. **55-56.** The membrane is supported by a continuous frame of rigid poles. Low vertices and summits are hinged. The whole structure is anchored to the ground in the low vertices. **54.** Since the preceding structure can be made hexagonal, similar units can be assembled together to cover a big area. Units are suggested as tents of simple erection. **57-58.** In this case the frame of poles was eliminated and substituted by a pole in each summit. The edge of the membrane is a tensile member. **59-60.** A variation of the preceding structure. The membrane is carried beyond the supports and anchored to the ground. Wrinkles caused by internal compressive stresses are seen on the periphery because one piece of elastic material was used. **61-62.** Scheme for a long span cover where the central part is open. Twelve summits and twelve low vertices. On the model, the form was generated with an inappropriate technique and the membrane is flat in the interior.



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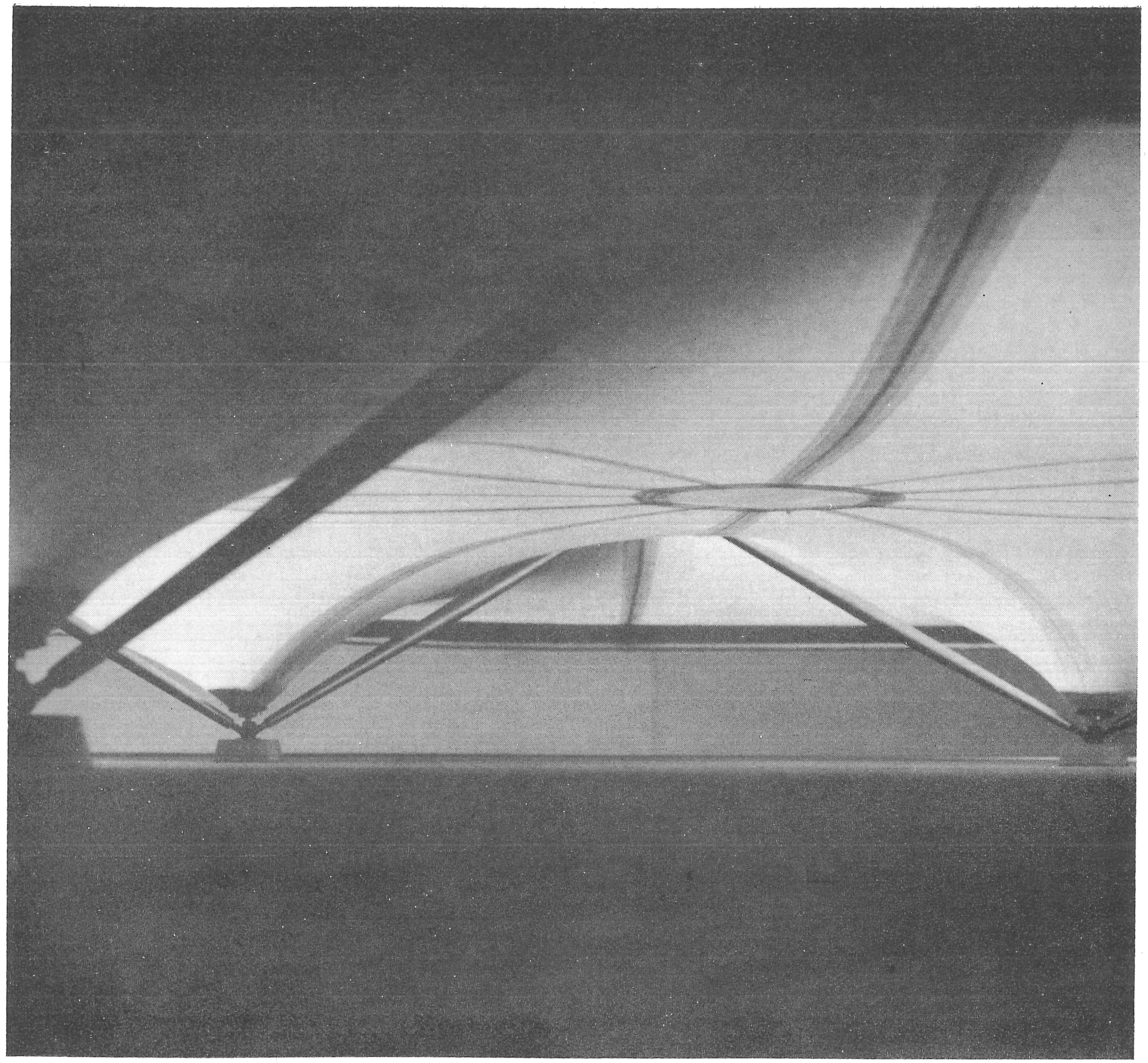
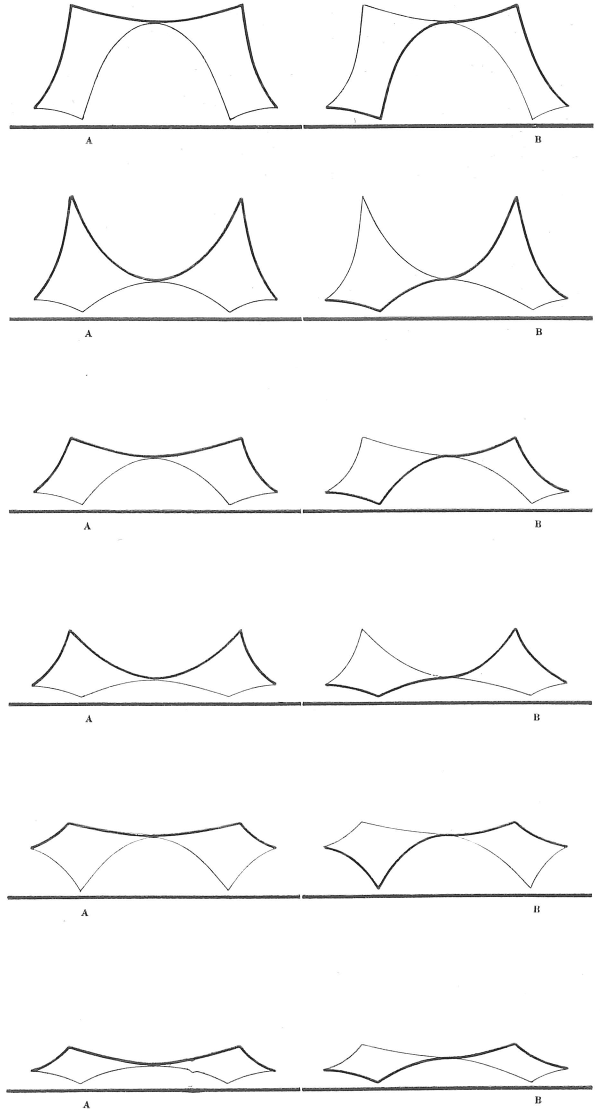


63. Interior view of a tensile structure.



64. Schematic sections showing variation achieved by vertical displacements of summits, ring, or center. Schemes A are six-support structures. Schemes B are five-support structures.

VI APPROACH TO THE STRESS ANALYSIS

REMARKS ON THE ANALYSIS OF STATIC BEHAVIOR AND OF INTERNAL STRESS DISTRIBUTION.[7]

The first theoretical studies on membranes (curved membranes of revolution without flexural rigidity) were made by G. Lame and E. Clapeyron prior to 1828. In 1833, they published a paper: *"Memoir sur l'Equilibre Interiour des Corps Solides Homogenes", Memoire Presente Par Divers Savants—Vol. IV.* ("Report on the Internal Equilibrium of Solid Homogeneous bodies"). However, only the last decades have seen the first practical applications in building of the most simple expressions of the membranal concept, especially through studies made by F. Dischinger and W. Flugge.

The most recent examples are particularly due to the statical intuition of a few ingenious designers. Some of these structures are of great interest because they are the result of experimental research on models carried on in specialized laboratories.

The application of the concept of "pure infinite membrane" is impossible in practice since there is a limit of flexural rigidity. Therefore, other factors must be considered such as those concerning edge perturbations and those of introduction of bending moments. As a consequence, the mathematical analysis becomes rather difficult.

Only spatial surfaces of elementary shape can be studied and analyzed to find with satisfactory ac-

[7] "The problems that an engineer has to face and solve are not always those to which we can apply analytical methods. Furthermore, the engineer cannot be stopped by the limitations that seem imposed by the state of our scientific knowledge and by the possibilities of calculation.

"He cannot stop, because he knows, and verifies every day, that beyond our possibilities of analytical investigation, exists a world of new possibilities, which are suggested by experience and still more often, by the simple observation and sensitive interpretation of nature's phenomena.

"On the other hand, the history of engineering offers us many examples in which the intuition of the designer has happily preceded the developments of the theory. Such developments have given later to the designer's intuition the assurance of a full and therefore more precious justification."

—Professor G. Colonnetti (President of the National Research Council—Italy) curacy their static behavior. Even for these shapes, (surfaces of revolution with vertical axis, translational surfaces with horizontal axis), the mathematical calculus is difficult. The field of other infinite surfaces is almost unknown, notwithstanding their structural advantages.

It can be seen, then, that in the case of the structures discussed here, the stress analysis presents a double difficulty: one, inherent to the shape itself, since the membrane is not an elementary known surface; the other due to the physical limits of the membrane that originates edge perturbations.

Therefore, it seem that an approach to the problem with an analytical-experimental method is unavoidable.

Before considering this approach, it is necessary to review some of the possibilities of variation of the structural system. They can be listed as follows:

Type I: Continuous membrane in canvas or sheet metal, supporting poles and rigid ring.

Type II: Network in cables, rods or wires, supporting poles and rigid ring.

Type III: Continuous membrane in reinforced concrete, supporting poles and rigid ring.

Type IV: Continuous membrane in reinforced concrete, self-supported, rigid ring. Obtained in the following manner: pre-stressing a network; pouring concrete; removing supports when concrete settles.

An approach to the stress analysis is being developed only on the continuous type, since a solution for this type can be applied to reticular structures after proper considerations.

It can be noted also that at this preliminary stage only structures of circular plan with symmetrical summits and low vertices have been included because these seem to be the simplest to study.

The analysis was approached in the following manner:

Initially, several calculations were made based upon different elementary hypotheses of structural behavior. These calculus of "maxima" have led to similar results. The purpose was to get only approximate quantitative magnitudes for a preliminary design (dimensions, proportions, forms).

The simplicity of the methods used was justified at the beginning, since rigorous mathematical stress analysis could scarcely be of value if it were based on dimensions that were not related to reality. In fact, the rigorous mathematical methods—useful as they can be—are, even in simple structures, only instruments of verification.

After this preliminary design approach, two methods of investigation follow:

A. Theoretical analysis of stress distribution in the membrane for required load conditions.

B. Experimental analysis to find the stress distribution in the membranal surface for the required load conditions, carried to complete rupture, to determine the overall static behavior. It must be pointed out that the overall behavior of the structure is often as important as the knowledge of stress distribution under normal load conditions, because it is the only way to know the real and ultimate possibilities of the structure.

Certainly, the two different methods (analytical and experimental) are both interesting because the first would have to be the interpretation of final experimental results. It seems opportune to remember Leonardo's statement that "a science cannot exist without experience," which often alone can answer the needs of practical reality. However, this conclusion does not minimize the role of theoretical search which frequently gives accurate results even before experimentation.

In the case of the typical structure, the theoretical analysis has led to an approach to a general expression of equilibrium for membranes with a central vertical axis that may be useful if resolved by finite differences. Consequently, it can give approximate solutions, compatible with the accuracy of the calculus.

Experimental analysis based on loading tests is carried on simultaneously, but only those acquainted with the procedures will understand the slowness of the progress.

AN APPROACH TO A GENERAL EXPRESSION OF EQUILIBRIUM FOR MEMBRANES WITH A CENTRAL VERTICAL AXIS

The following notes develop an approach for theoretical analysis of any type of membrane with a central vertical axis and not only for surfaces of revolution, which are particular cases of this theory.

This is, naturally, a first approach that may have further development when applied to a specific membranal surface. In order to avoid too difficult mathematical expressions, even for the limited conditions, it is more convenient to use the particular surface in each case.

In fact, this theoretical analysis was developed to study the typical structures presented in this article; but the pertinent application is not treated here and will be discussed in another article.

THE SYSTEM OF COORDINATES. Assume a fixed system of three rectangular coordinates axes OX, OY, OZ, where OZ is the central vertical axis of the membranal surface in question. (fig. 65)

The coordinates of any point of the surface can be expressed in a parametric way:

$$x = R \sin \phi \sin \theta$$
$$y = R \sin \phi \cos \theta$$
$$z = R \cos \phi$$

The variable terms are R, ϕ, θ. In other words, any point of the selected surface can be expressed in terms of the variable radius R and angles ϕ, θ.

Assume, in addition, a secondary system of three rectangular coordinate axes with a variable position in space. This secondary system is determined in the following manner:

Consider a point P (x, y, z) on the membrane surface, at a distance $R \sin \phi$ from the axis OZ. This point lies also in the surface of a right circular cylinder with vertical axis coincident with Z and a radius $R \sin \phi$. Consider, finally, the vertical plane through OZ and the point P.

Two curves are determined, then, on the membrane surface: one, the intersection of membrane and cylinder; the other, intersection of membrane and vertical plane.

These two curves have in point P, two tangent lines:

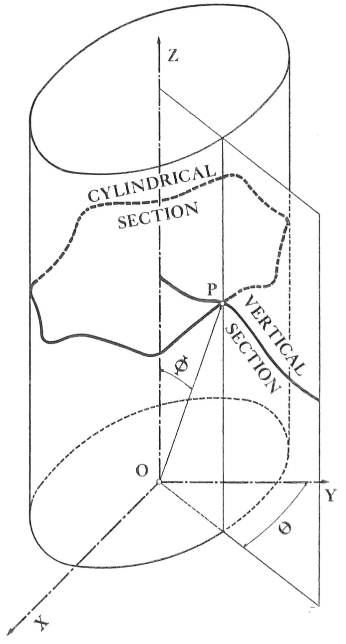
l = tangent contained on a plane tangent to the cylinder.

m = tangent contained on the vertical plane.

Evidently, these two tangents are perpendicular since they lie in perpendicular planes.

A third straight line n perpendicular to both tangents completes the variable system of rectangular coordinate axes Pl, Pm, Pn.

Any point on the membrane surface can be defined in this manner—through the intersection of two selected curves. Consequently, the correspondent tangents at the point can be defined also. In other words, two axes of the variable coordinate system are tangent to the surface at any given point.



65. The system of coordinates.

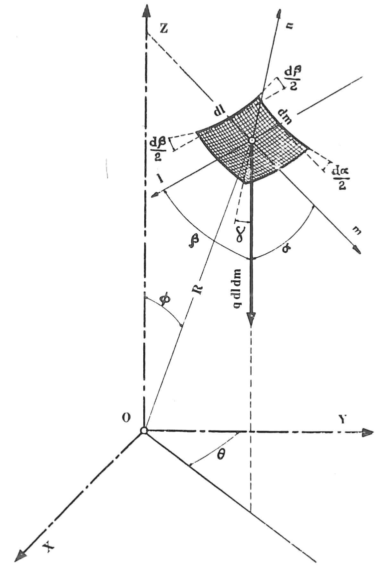
GEOMETRICAL CONSIDERATIONS AND CORRESPONDING MATHEMATICAL EXPRESSIONS. A generical point of the membranal surface is defined by R, ϕ, θ, in relation with the fixed system of coordinate axis OX, OY, OZ. (fig. 66-69)

Around this point, consider an infinitely small surface element, with sides dl, dm, respectively parallel to axes Pl and Pm. The orientation in space of this element dl, dm is determined by the angle α (formed by vertical line and axis m) and the angle β (formed by a vertical line and axis l).

Knowing these two angles, it is easy to find the value of angle γ (formed by the vertical and perpendicular lines to the element $dl\ dm$ in the point considered.)

However, it should be observed that dl can have a variation according to the axis m, either with reference to the dimension or the orientation, similarly for dm, according to l.

Consequently, it is necessary to define the expressions of both of the two variations; that is to say, $\delta(dl)$ and $\delta(dm)$.



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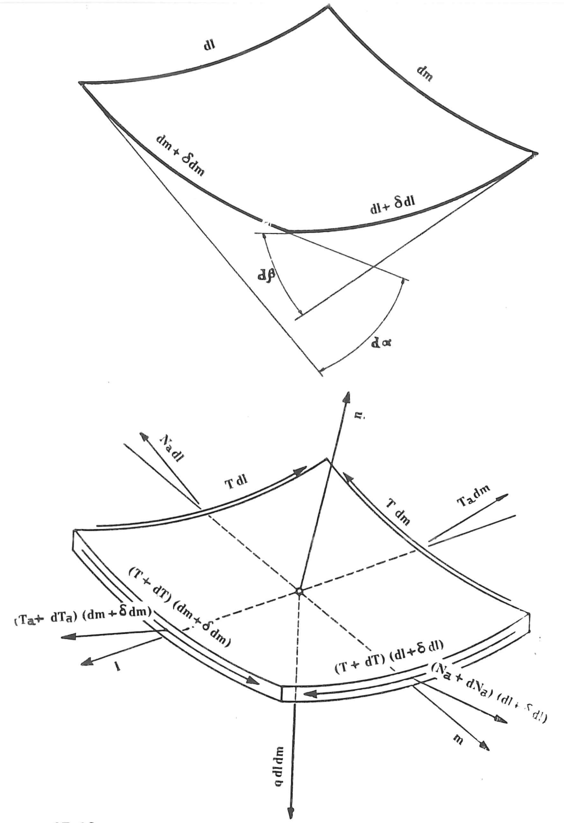

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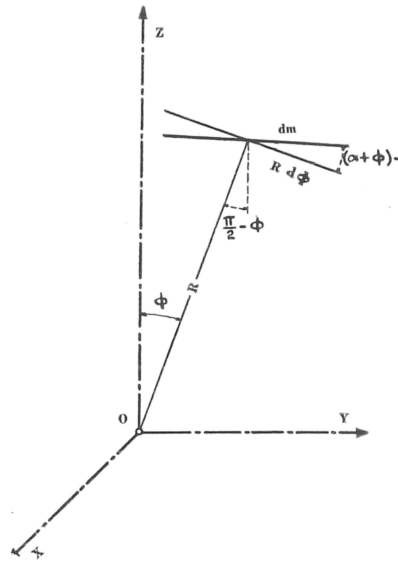


69.

The size dl can be expressed by the following relation:

$$dl = \frac{\sin\phi}{\sin\beta} R\, d\theta$$

and consequently:

$$\delta(dl) = \frac{\partial}{\partial m}(dl)\, dm = \frac{\partial R}{\partial m} dm \frac{\sin\phi}{\sin\beta} d\theta$$

$$+ R \frac{\cos\phi}{\sin\beta} \frac{\partial\phi}{\partial m} dm\, d\theta - R \frac{\sin\phi \cos\beta}{\sin^2\beta} \frac{\partial\beta}{\partial m} dm\, d\theta$$

and similarly for dm:

$$dm = \frac{R}{\sin(\alpha+\phi)} d\phi$$

and consequently:

$$\delta(dm) = \frac{\partial}{\partial l}(dm)\, dl = \left[\frac{\partial R}{\partial l} \frac{1}{\sin(\alpha+\phi)} \right.$$

$$\left. - R \frac{\cos(\alpha+\phi)\left(\frac{\partial\alpha}{\partial l} + \frac{\partial\phi}{\partial l}\right)}{\sin^2(\alpha+\phi)} \right] dl\, d\phi$$

Values of $d\theta$ and $d\phi$ are:

$$d\theta = \frac{1}{R} \frac{\sin\beta}{\sin\phi} dl$$

$$d\phi = \frac{1}{R} \sin(\alpha+\phi)\, dm$$

substituting:

$$\delta(dl) = \frac{\partial}{\partial m}(dl)\, dm = \left[\frac{1}{R} \frac{\partial R}{\partial m} \right.$$

$$\left. + \frac{1}{R} \cot\phi \sin(\alpha+\phi) - \cot\beta \frac{\partial\beta}{\partial m} \right] dm\, dl$$

$$\delta(dm) = \frac{\partial}{\partial l}(dm)\, dl = \left[\frac{1}{R} \frac{\partial R}{\partial l} \right.$$

$$\left. - \left(\frac{\partial\alpha}{\partial l} + \frac{\partial\phi}{\partial l} \right) \cot(\alpha+\phi) \right] dm\, dl$$

CONSIDERATIONS OF EQUILIBRIUM.

After the preceding preliminary considerations, the equilibrium of the surface element of the membrane, $dl\ dm$, can be analyzed to determine the equilibrium expressions of all the forces acting on the element resolved into three perpendicular directions.

External loads are considered vertical surface loads.

Consequently, assuming a uniform thickness of the membrane, the unit dead load is constant at any point of the membrane. For other load distributions, it is supposed that the law of variation of the unit load is given. Nevertheless, it is often possible to consider such external loads as a uniform load distribution without a great error.

The forces acting on the infinitesimal element of the surface are:

1) the external vertical load: $q\ dl\ dm$.

2) the perpendicular force $N_a\ dl$, acting on dl, according to direction m.

3) the perpendicular force $T_a\ dl$, acting on dm, according to direction l.

4) the shear forces Tdl and Tdm, according to the two directions l, m,

5) also, the variations of the mentioned forces, according l and m.

Bending moments have been neglected.

EQUILIBRIUM WITH RESPECT TO THE VERTICAL (THE DIRECTION OF EXTERNAL LOAD).

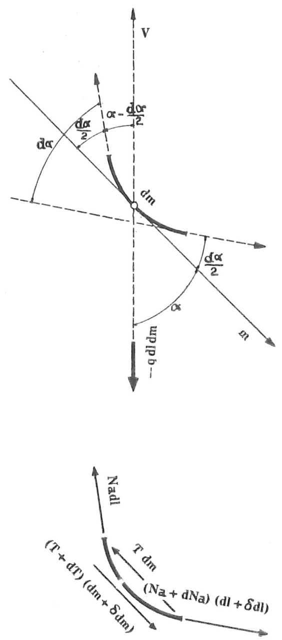
In order to write the equilibrium equation for vertical displacement, two sections are considered, determined by vertical planes through m, v and l v. Figures 70-71 show the forces and correspondent angles with the vertical line V

$$N_a\, dl \cos\left(\alpha - \frac{d\alpha}{2}\right) - \left(N_a + \frac{\partial N_a}{\partial m} dm\right)$$

$$(dl + \delta dl)\cos\left(\alpha + \frac{d\alpha}{2}\right) + T_a\, dm \cos\left(\beta - \frac{d\beta}{2}\right)$$

$$-\left(T_a + \frac{\partial T_a}{\partial l} dl\right)\left(dm + \delta dm\right)\cos\left(\beta + \frac{d\beta}{2}\right)$$

$$+ T\, dm \cos\alpha - \left(T + \frac{\partial T}{\partial l} dl\right)\left(dm + \delta dm\right)\cos\alpha$$



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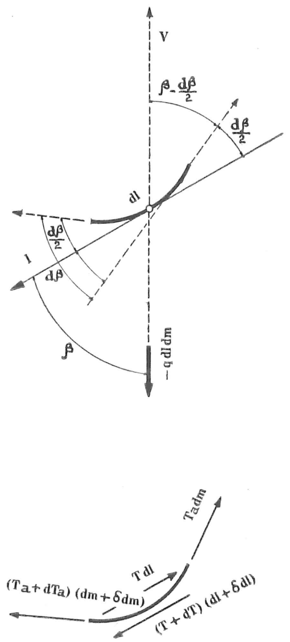

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$$+ T\, dl \cos\beta - \left(T + \frac{\partial T}{\partial m} dm\right)\left(dl + \delta dl\right)\cos\beta$$

$$- q\, dl\, dm = o$$

Developing and neglecting the terms of third and fourth order:

$$N_a \sin\alpha\, dl\, d\alpha - N_a\, \delta dl \cos\alpha - \frac{\partial N_a}{\partial m} dl\, dm \cos\alpha$$

$$+ T_a \sin\beta\, d\beta - T_a\, \delta dm \cos\beta - \frac{\partial T_a}{\partial l} dl\, dm \cos\beta$$

$$- T\, \delta dm \cos\alpha - \frac{\partial T}{\partial l} dm\, dl \cos\alpha - T\, \delta dl \cos\beta$$

$$- \frac{\partial T}{\partial m} dm\, dl \cos\beta - q\, dl\, dm = o$$

and dividing by $dl\ dm$ and substituting values of $\delta(dl)$ and $\delta(dm)$

$$N_a \cos\alpha \left[\frac{1}{R}\frac{\partial R}{\partial m} + \frac{1}{R}\cot\phi \sin(\alpha + \phi)\right.$$

$$\left. - \cot\beta \frac{\partial \beta}{\partial m} - \tan\alpha \frac{\partial \alpha}{\partial m}\right] + \frac{\partial N_a}{\partial m}\cos\alpha$$

$$+ T_a \cos\beta \left[\frac{1}{R}\frac{\partial R}{\partial l} - \left(\frac{\partial \alpha}{\partial l} + \frac{\partial \phi}{\partial l}\right)\cot(\alpha + \phi)\right.$$

$$\left. - \tan\beta \frac{\partial \beta}{\partial m}\right] + \frac{\partial T_a}{\partial l}\cos\beta + T\cos\alpha \left[\frac{1}{R}\frac{\partial R}{\partial l}\right.$$

$$\left. - \left(\frac{\partial \alpha}{\partial l} + \frac{\partial \phi}{\partial l}\right)\cot(\alpha + \phi)\right] + \frac{\partial T}{\partial l}\cos\alpha$$

$$+ T\cos\beta \left[\frac{1}{R}\frac{\partial R}{\partial m} + \frac{1}{R}\cot\phi \sin(\alpha + \phi)\right.$$

$$\left. - \cot\beta \frac{\partial \beta}{\partial m}\right] + \frac{\partial T}{\partial m}\cos\beta = -q$$

EQUILIBRIUM WITH RESPECT TO THE HORIZONTAL STRAIGHT LINE LYING IN THE VERTICAL PLANE PASSING THROUGH Z, m, V.

This straight line is evidently perpendicular to V, that is to say, to the direction of the loads. Figures 72-76 may help the reader to see the intersecting forces.

$$- T_a\, dm \sin\beta \sin\frac{d\theta}{2} - \left(T_a + \frac{\partial T_a}{\partial l} dl\right)$$

$$(dm + \delta dm)\sin\beta \sin\frac{d\theta}{2}$$

$$- N_a\, dl \cos\left[\frac{\pi}{2} - \left(\alpha - \frac{d\alpha}{2}\right)\right] + \left(N_a + \frac{\partial N_a}{\partial m} dm\right)$$

$$(dl + \delta dl)\cos\left[\frac{\pi}{2} - \left(\alpha + \frac{d\alpha}{2}\right)\right]$$

$$- T\, dm \cos\left(\frac{\pi}{2} - \alpha\right) + \left(T + \frac{\partial T}{\partial l} dl\right)$$

$$(dm + \delta dm)\cos\left(\frac{\pi}{2} - \alpha\right) = o$$

developing and neglecting the terms of third and fourth order and dividing by dl, dm:

$$- T_a \sin\beta \frac{d\theta}{dl} + N_a \cos\alpha \frac{\partial \alpha}{\partial m} + N_a \sin\alpha \frac{\delta \partial l}{dl\, dm}$$

$$+ \frac{\partial N_a}{\partial m}\sin\alpha + T\sin\alpha \frac{\delta dm}{dl\, dm} + \frac{\partial T}{\partial l}\sin\alpha = o$$

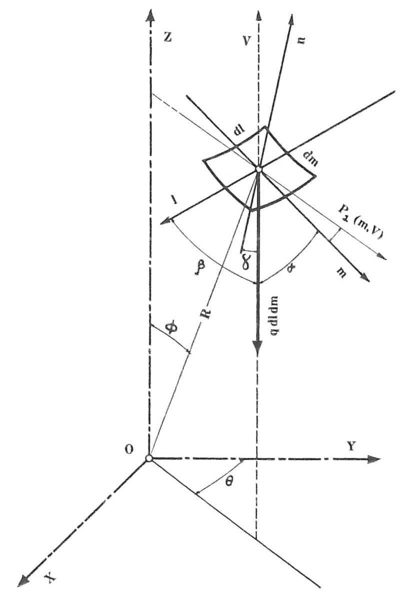
Substituting the values of $d\theta$ and $\delta(dl)$, $\delta(dm)$

$$N_a \left[\frac{1}{R}\frac{\partial R}{\partial m} + \frac{1}{R}\cot\phi \sin(\alpha + \phi)\right.$$

$$\left. - \cot\beta \frac{\partial \beta}{\partial m} + \cot\alpha \frac{\partial \alpha}{\partial m}\right]$$

$$+ \frac{\partial N_a}{\partial m} - \frac{T_a}{R}\frac{\sin^2\beta}{\sin\phi \sin\alpha} + T\left[\frac{1}{R}\frac{\partial R}{\partial l}\right.$$

$$\left. - \left(\frac{\partial \alpha}{\partial l} + \frac{\partial \phi}{\partial l}\right)\cot(\alpha + \phi)\right] + \frac{\partial T}{\partial l} = o$$



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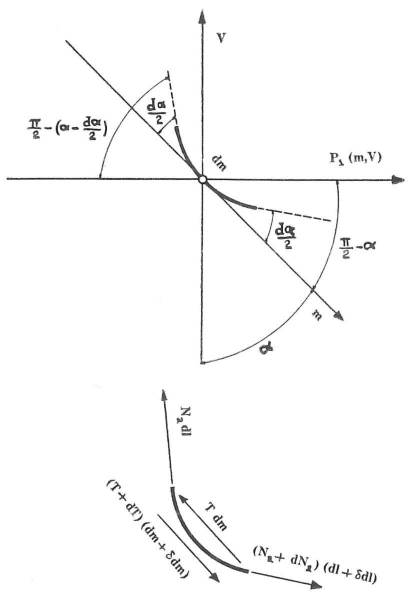

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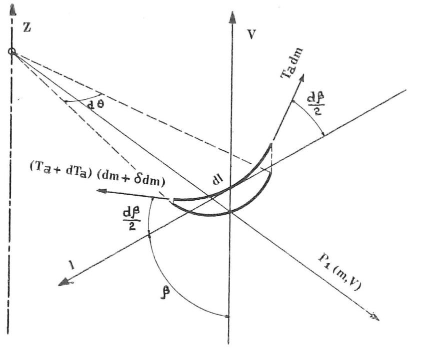
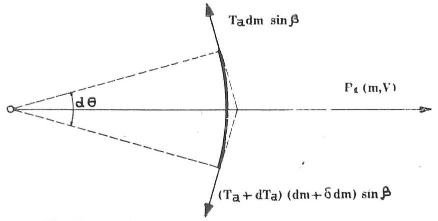
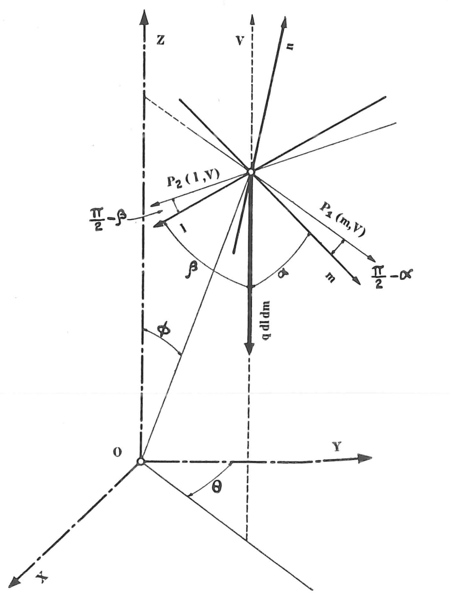


75. Projecting on the horizontal plane.



76. Horizontal projection.



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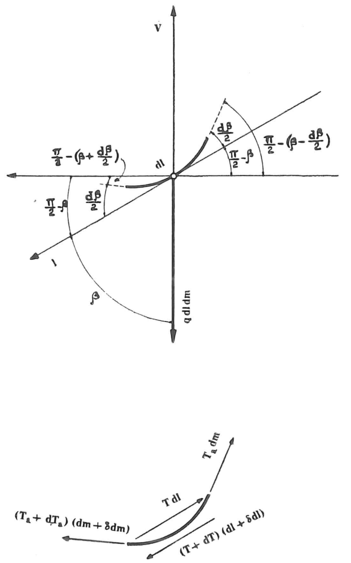

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EQUILIBRIUM WITH RESPECT TO THE HORIZONTAL STRAIGHT LINE LYING ON THE VERTICAL PLANE PASSING THROUGH V, l AND PERPENDICULAR TO THE STRAIGHT LINE.

Figures 77-78 may help the reader to see the intersecting forces.

$$-T_\alpha\, dm \cos\left[\frac{\pi}{2}-\left(\beta-\frac{d\beta}{2}\right)\right]+\left(T_\alpha+\frac{\partial T_\alpha}{\partial l}\, dl\right)$$

$$(dm+\delta dm)\cos\left[\frac{\pi}{2}-\left(\beta+\frac{d\beta}{2}\right)\right]$$

$$-T\, dl \cos\left(\frac{\pi}{2}-\beta\right)+\left(T+\frac{\partial T}{\partial m}\, dm\right)$$

$$(dl+\delta dl)\cos\left(\frac{\pi}{2}-\beta\right)=0$$

Developing and neglecting the terms of third and fourth order:

$$T_\alpha \cos\beta\, d\beta\, dm+T_\alpha\, \delta dm \sin\beta+\frac{\partial T_\alpha}{\partial l}\sin\beta\, dl\, dm$$

$$+T\sin\beta\, \delta dl+\frac{\partial T}{\partial m}\sin\beta\, dl\, dm=0$$

Substituting the values of $\delta(dm)$, $\delta(dl)$, and dividing by $\sin\beta\, dl\, dm$:

$$T_\alpha\left[\frac{1}{R}\frac{\partial R}{\partial l}-\left(\frac{\partial\alpha}{\partial l}+\frac{\partial\phi}{\partial l}\right)\cot(\alpha+\phi)\right.$$

$$\left.+\cot\beta\frac{\partial\beta}{\partial l}\right]+\frac{\partial T_\alpha}{\partial l}+T\left[\frac{1}{R}\frac{\partial R}{\partial m}\right.$$

$$\left.+\frac{1}{R}\cot\phi\sin(\alpha+\phi)-\cot\beta\frac{\partial\beta}{\partial m}\right]+\frac{\partial T}{\partial m}=0$$

Collecting the three equations and calling:

$$K=\frac{1}{R}\frac{\partial R}{\partial m}+\frac{1}{R}\cot\phi\sin(\alpha+\phi)-\cot\beta\frac{\partial\beta}{\partial m}$$

$$H=\frac{1}{R}\frac{\partial R}{\partial l}-\left(\frac{\partial\alpha}{\partial l}+\frac{\partial\phi}{\partial l}\right)\cot(\alpha+\phi)$$

The final system results:

$$\left[K-\tan\alpha\frac{\partial\alpha}{\partial m}\right]N_\alpha\cos\alpha+\frac{\partial N_\alpha}{\partial m}\cos\alpha$$

$$+\left[H-\tan\beta\frac{\partial\beta}{\partial m}\right]T_\alpha\cos\beta+\frac{\partial T_\alpha}{\partial l}\cos\beta$$

$$+\left[H\cos\alpha+K\cos\beta\right]T+\frac{\partial T}{\partial l}\cos\alpha$$

$$+\frac{\partial T}{\partial m}\cos\beta=-q$$

$$\left[K+\cot\alpha\frac{\partial\alpha}{\partial m}\right]N_\alpha+\frac{\partial N_\alpha}{\partial m}-\left[\frac{1}{R}\cdot\frac{\sin^2\beta}{\sin\phi\sin\alpha}\right]T_\alpha$$

$$+HT+\frac{\partial T}{\partial l}=0$$

$$\left[H+\cot\beta\frac{\partial\beta}{\partial l}\right]T_\alpha+\frac{\partial T_\alpha}{\partial l}+KT+\frac{\partial T}{\partial m}=0$$

APPLICATION OF THE THREE EQUATIONS OF THE FINAL SYSTEM TO A SURFACE OF REVOLUTION.

Case of spheric dome of uniform thickness, under dead loads:

$$\alpha=\frac{\pi}{2}-\phi;\ \beta=\frac{\pi}{2}=\text{constant};\ R=\text{constant}$$

$$d\alpha=-d\phi;\ \frac{d\beta}{dm}=0;\ dm=R\,d\phi$$

$$\frac{\partial T_\alpha}{\partial l}=0$$

The expressions of K and H become:

$$K=\frac{\cot\phi\sin\left[\frac{\pi}{2}-\phi+\phi\right]}{R}=\frac{\cot\phi}{R}$$

$$H=0$$

Consequently, the system is transformed to:

$$2\cos\phi\, N_\alpha+\frac{dN_\alpha}{dy}\sin\phi+\frac{\partial T}{\partial l}R\sin\phi=-q\,R$$

$$(\cos^2\phi-\sin^2\phi)\,N_\alpha+\frac{dN_\alpha}{d\phi}\sin\phi-T_\alpha$$

$$+R\frac{\partial T}{\partial l}\sin\phi\cos\phi=0$$

$$\frac{\cot\phi}{R}T+\frac{\partial T}{\partial m}=0$$

The solution of the third equation is:
$T=-c\sin\phi$

And substituting the same equation, it is determined the value of $c=0$ and consequently:

$$T=\frac{\partial T}{\partial m}=\frac{\partial T}{\partial l}=0$$

With this value, it is possible to express the system by:

$$2\cos\phi\, N_\alpha+\frac{dN_\alpha}{d\phi}\sin\phi=-q\,R$$

$$(\cos^2\phi-\sin\phi)\,N_\alpha+\frac{dN_\alpha}{d\phi}\sin\phi\cos\phi-T_\alpha=0$$

The two solutions are:

$$N_\alpha=-\frac{qR}{1+\cos\phi}$$

$$T_\alpha=qR\frac{(1-\cos\phi-\cos^2\phi)}{1+\cos\phi}$$

in accord with the membranal theory for surfaces of revolution.

APPENDIX

THE APPLICATION OF THE THEORY OF SIMILITUDE TO THE INVESTIGATION OF STATIC BEHAVIOR THROUGH MODELS.

Since the analysis of static behavior of the structures is carried out experimentally on models, values obtained must be extrapolated from models to full size.

These changes in scale are based upon the theory of similitude. A simple explanation of the theory is attempted in these brief notes for the benefit of those students interested in the procedure.

The expression of a physical phenomena can be characterized through some fundamental greatnesses called "dimensions", which define the internal structure of any physical expression and permit the establishment of relations with others.

In the field of statics it is enough to choose length, mass and time, as fundamental dimensions, which are called, respectively, *L*, *M*, *T*.

In other words, the dimensional analysis is indispensable in order to define the "unit of measure" with which physical quantities can be expressed.

It is evident that with the unit of weight, distance can not be measured, because weight and length do not have the same dimensional quality.

Therefore, the dimensional analysis defines the category to which a certain physical expression pertains.

For example: the kinetic energy of a body with a mass *M* and a velocity *V* is usually expressed by $\Sigma = \frac{1}{2} MV^2$. The mass is a fundamental assumed dimension and velocity is the ratio between a length and a time, consequently it is possible to write dimensionally:

$$[\Sigma] = \left[M \left(\frac{L}{T} \right)^2 \right] = [ML^2 T^{-2}]$$

Work is a product of a force *F* by a length *S;* that is to say: $W = F \times S$. But force is the product of a mass by an acceleration, acceleration being a ratio between velocity and time. Length is a fundamental dimension. Dimensionally it is possible to write:

$$[W] = [M \frac{L}{T^2} L] = [M L^2 T^{-2}]$$

Through the comparison of the two dimensional expressions, it is easy to recognize that the internal nature of the kinetic energy and of the work is the same.

The preceding example shows how simple it is to find the internal nature of a physical quantity through dimensional analysis. Therefore, it is easy now to understand why a generic physical quantity *Q* can be expressed dimensionally by:

$$[Q] = [M^a L^b T^c]$$

in which a, b, c have different numerical values according to the considered quantity. For example:

LENGTHS:

$[L] = [L]$ $a = 0;\ b = 1;\ c = 0$

AREAS.

$[A] = [L^2]$ $a = 0;\ b = 2;\ c = 0$

VOLUMES:

$[V] = [L^3]$ $a = 0;\ b = 3;\ c = 0$

STATIC MOMENT:

$[AL] = [L^3]$ $a = 0;\ b = 3;\ c = 0$

SECTION MODULUS:

$[S] = [L^3]$ $a = 0;\ b = 3;\ c = 0$

MOMENT OF INERTIA:

$[I] = [L^4]$ $a = 0;\ b = 4;\ c = 0$

DENSITY:

$[\frac{M}{V}] = [ML^{-3}]$ $a = 1;\ b = -3;\ c = 0$

VELOCITY:

$[\frac{L}{T}] = [LT^{-1}]$ $a = 0;\ b = 1;\ c = -1$

ACCELERATION:

$[a] = [\frac{L}{T^2}] = [LT^{-2}]$ $a = 0;\ b = 1;\ c = -2$

FORCES:

$[Ma] = [MLT^{-2}]$ $a = 1;\ b = 1;\ c = -2$

FORCES PER UNIT OF LENGTH:

$[\frac{Ma}{L}] = [MT^{-2}]$ $a = 1;\ b = 0;\ c = -2$

FORCES PER UNIT OF AREA (UNIT STRESS):

$[\frac{Ma}{L^2}] = [ML^{-1} T^{-2}]$ $a = 1;\ b = -1;\ c = -2$

MODULUS OF ELASTICITY:

$[E] = [\frac{Ma}{L^2}] = [ML^{-1} T^{-2}]$ $a = 1;\ b = -1;\ c = -2$

After this dimensional review of the principal expressions of statics, it is possible to consider the relations between a real structure and a similar model not necessarily of the same material. In other words, relations can be established between greatness measured on the model and the correspondent on the structure and vice versa.

A greatness on the model can be expressed dimensionally by:

$$q_m = [M^a L^b T^c]$$

and the corresponding greatness in the real structure:

$$Q_R = [(mM)^a (lL)^b (tT)^c]$$

in which *l*, *m*, *t* are non-dimensional factors ,that is to say, numbers.

The ratio between the two greatnesses gives the fundamental relation:

$$Q_R = [m^a l^b t^c]\ q_m$$

Where, evidently, the expression between parentheses is a non-dimensional factor, because the dimensions of Q_R and q_m are the same.

A PRACTICAL EXAMPLE:

Model in celluloid in scale 1:10 of the real structure in reinforced concrete.

MODEL

Modulus of elasticity of celluloid:

$E_m = 300{,}000$ lbs./sq. inch

Density of celluloid:

$D_m = \frac{50}{g}$ lbs mass/cu. ft.

(g = acceleration of gravity)

REAL STRUCTURE

Modulus of elasticity of concrete:

$E_R = 3{,}000{,}000$ lbs./sq. inch

Density of concrete:

$D_R = \frac{150}{g}$ lbs. mass/ cu. ft.

Writing the dimensional expressions for the two factors of the model and of the real structure:

$$\begin{cases} E_m = [ML^{-1}\ T^{-2}] = 300{,}000 \\ E_R = [(mM)\ (lL)^{-1}\ (tT)^{-2}] = 3{,}000{,}000 \\ a = 1;\ b = -1;\ c = -2 \end{cases}$$

$$\begin{cases} D_m = [ML^{-3}] = \dfrac{50}{g} \\ D_R = [(mM)\ (lL)^{-3}] = \dfrac{150}{g} \\ a = 1;\ b = -3;\ c = 0 \end{cases}$$

The ratio between the two moduli of elasticity gives:

$$\frac{E_R}{Em} = \frac{3{,}000{,}000}{300{,}000} = 10 = m\ l^{-1}\ t^{-2}$$

The ratio between the two densities gives:

$$\frac{D_R}{D_m} = \frac{150}{50} = 3 = ml^{-3}$$

Through this second equation results:

$$m = 3l^3$$

Which, substituted in the first equation, gives:

$$3l^3\ l^{-1}\ t^{-2} = 10$$

That is:

$$t = \sqrt{3/10}\ l = (3/10)^{½}\ l$$

Remembering the ratio of the scale of length $L_R = 10\ L_m$ therefore: $l = 10$ consequently the final relation is:

$$Q_R = [m^a\ l^b\ t^c]\ q_m = [(cl^3)^a\ l^b\ (3/10)^{c/2}\ l^c]\ q_m =$$

$$= [\ 3^a \times (3/10)^{c/2}]\ 10^{3a+b+c} \times q_m$$

The above relation is for the particular considered case. Consequently, if the static elements q_m of the celluloid model have been measured through an experimental analysis, the correspondent values Q_R of the real structure in reinforced concrete can be immediately determined.

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COVER: INTERIOR VIEW OF A TENSILE STRUCTURE.
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