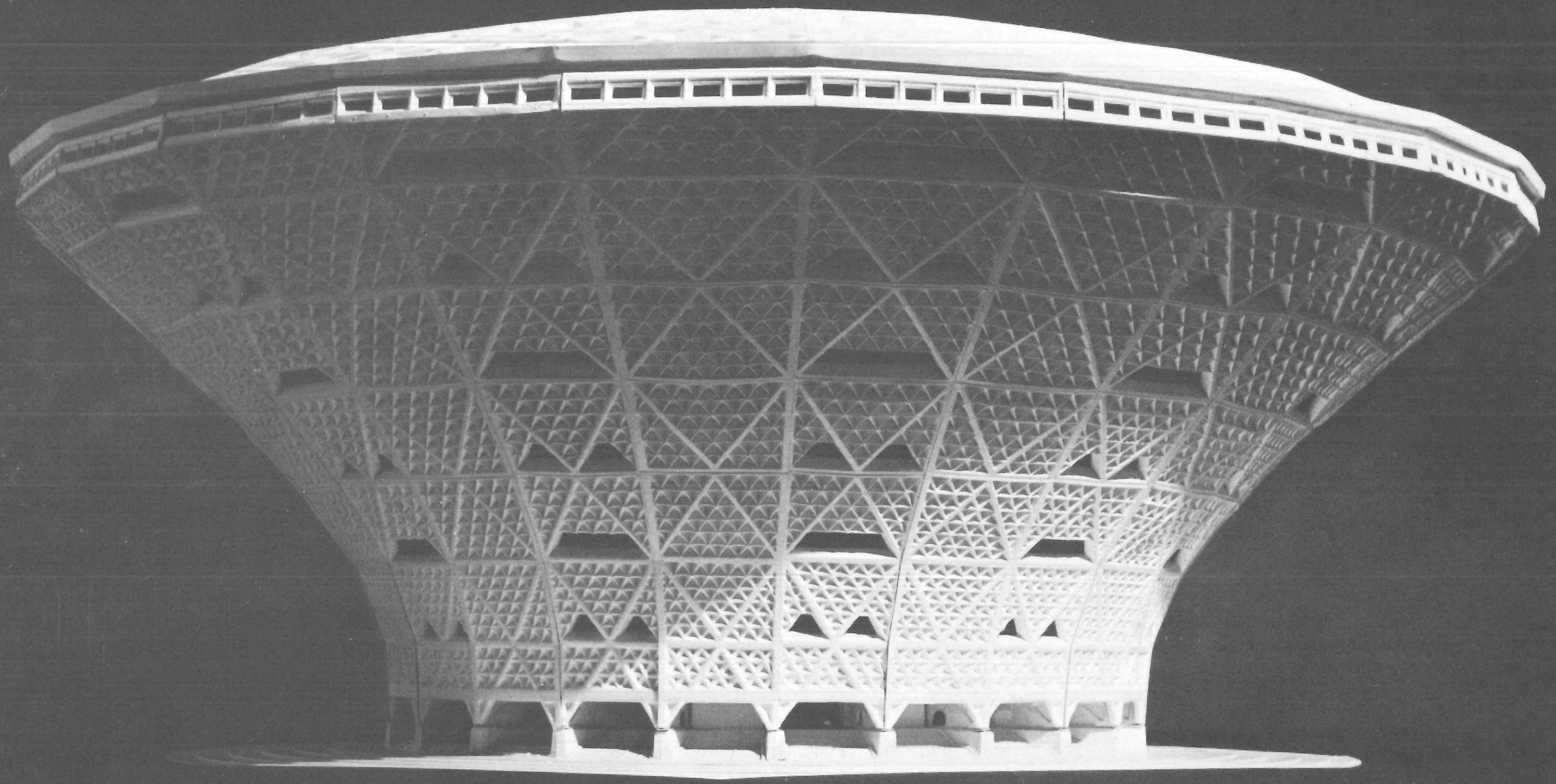
HORACIO CAMINOS

A THEME CENTER FOR A WORLD'S FAIR

TWO SURFACES OF REVOLUTION

HYPERBOLOID OF REVOLUTION OF ONE SHEET—CIRCULAR OPEN RING TORUS



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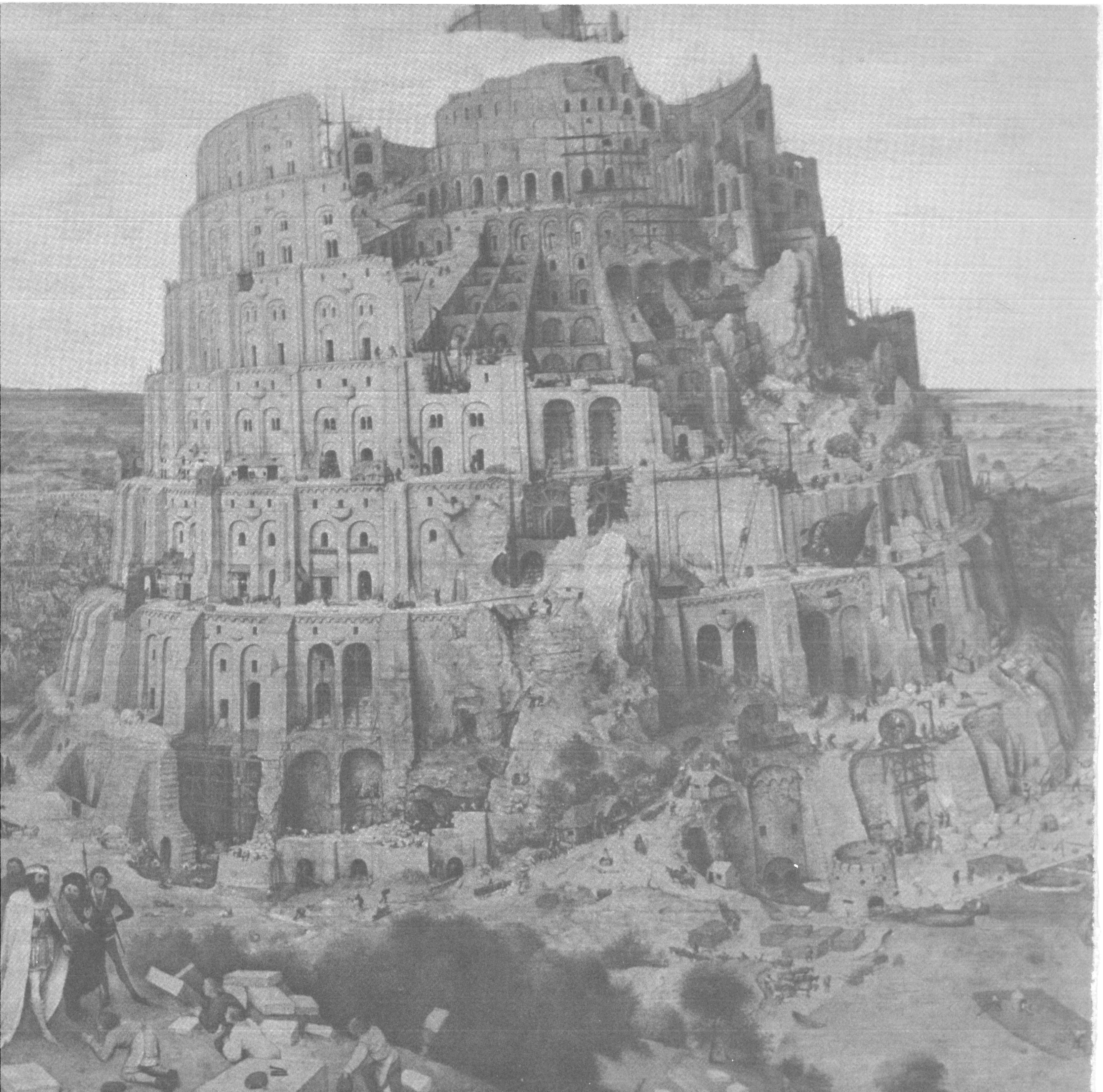

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INTRODUCTION

TWO STUDIES of different nature are presented here. The first one is a schematic description of a building for a N World's Fair which attempts to formulate a set of questions and to evolve a set of assumptions. It includes three similar projects that are variations of the same basic limitations. They are recorded in notes, drawings and models.**

The second one, on the hyperboloid and torus, is an elementary compilation of geometrical information about two surfaces that may have application in the construction of buildings. It was prepared mainly with the purpose of determining different manners of dividing the surfaces and subsequently of defining them with linear straight segments. In both cases the simplest method of plane sections was used. The presentation of the two surfaces follows the same sequence: a brief description of their geometrical properties as well as some algebraic expressions; generation; plane sections; rotation and translation of these sections; division of the surfaces in triangles and/or diamonds. At the end of each set of drawings a few photographs are included showing the generation of the surfaces by rotation of lines and planes. The last technique of study and representation deserves indeed a more thorough consideration than the one given on these pages.

These studies are presented together because they were contemporarily developed as parts of one problem. They retrace a long, although sporadic, period of incubation. They were sketched at length during the Spring of 1959. It was not until the Fall of 1960 that they had the opportunity to be prolixly developed with the cooperation of the students of the Fifth Year Architectural Design Course. Finally, models were completed during the Summer of 1961.

The reproductions on the back cover, as well as the fragments of prints accompanying three plates, are from paintings and engravings by Pieter Bruegel The Elder. They were not prepared for this publication and even the names under which such pieces are known may not bear a resemblance with the Theme Center: *The Tower of Babel, Elck or Everyman, Pride, The Fête of Fools.* Still they seem to provide the atmosphere that is lacking in the rest of the drawings: the construction industry at work; the exhibitors installing their paraphernalia; the pavilions and buildings that rise everywhere nowadays; finally, the festival at full blast.

HORACIO CAMINOS

Raleigh, December 12, 1961

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A THEME CENTER FOR A N** WORLD'S FAIR

Within the frame of an overall program for the Fair, the theme center building has been the particular subject of study. These brief notes summarize the main design limitations and assumptions.

PURPOSES

World's fairs have many purposes. These can range from promoting politics to providing diversion. Curiously enough the two extremes meet forming a circle which may include commercial, social, and cultural purposes.

In attracting hundreds of thousands of visitors from all over, they may aid in establishing communication among peoples. Generally, fairs have been organized on the basis of temporary exhibitions. Nevertheless since a great deal of energy, ingenuity, time, and money are involved, they could be planned on more permanent bases.

MAGNITUDES

The Great Exhibition—London 1851—Total area of the exhibition in one building (Crystal Palace) 800,000 square feet.

International Exhibition—Paris 1867—Total area of the exhibition in one building 1,800,000 square feet.

International Exhibition—Paris 1889—Area of main building (Galerie des Machines) 700,000 square feet (other buildings of the exhibition are not included).

Projected attendance figures for the **1964-65 New York World's Fair** give a total of 70,000,000 people in one year, with over 500,000 as the daily peak.

SITE

1200 acres of marshy land on the outskirts of N** City. The site has convenient expressway, railroad, and subway approaches. The site is developed as a Recreation Park. The Fair itself occupies 600 acres of the total area. Buildings require approximately 5,000,000 square feet of enclosed space. Parking areas for 20,000 cars. The Fair will be open for one year. Most of the constructions for the exhibits will be of a strictly temporary nature to be removed after the Fair closes. Only the Theme Center will remain as a permanent building to be used as an Arts and Sciences Center for the City with permanent as well as transitory displays. The temporary buildings are for the following exhibitions: city, states, local and foreign private industries, and also for general services, administration, etc. The International Exhibition is located in the Theme Center.

MAIN REQUIREMENTS OF THE THEME CENTER BUILDING

The Theme Center will be visited by a mass of people, yet this mass is composed of individual human beings of different races, ages, interests, backgrounds, activities, credos, etc., etc.

The Theme Center will accommodate different kinds of exhibitions and paraphernalia from different countries with different styles and standards of living.

The Theme Center Building and a Plaza are the core of the Fair. People will approach the plaza and building either walking or using low-speed vehicles such as tractor trains, electric carts, moving sidewalks, etc. Service and trucks will have an underground connection.

Approximate distribution of areas: Exhibitions and public spaces 600,000 square feet. Public Service and vertical circulation, 200,000 square feet. Mechanical and building services, 80,000 square feet. Maximum occupancy of the building, 25,000 persons.

Exhibitions and public spaces include: International Exhibitions; Restaurants and cafeterias for 2,000 people; Bars and Cafes for 1,000 people; Night Clubs for entertainment, dining, and dancing for 400 people; Lounges for 3,000 people; Observation decks.

Public Services include: Coat and rest rooms, kitchens, storage, administration and information offices, travel, post offices, police, first aid, etc.

Additional requirements of the building: a) To predominate over the fairgrounds and be clearly individualized from a distance; **b)** To define the Plaza; **c)** To provide spaces from which the Fair itself as well as the skyline of the City can be viewed; **d)** To offer a maximum unencumbered space free of columns, stairs, elevators, mechanical cores, for a maximum flexibility; **f)** To provide a simple, direct system of circulation; **g)** To comply with the regulations of the local Building Code.

DESIGN

The basic outline of the building was established based upon the preceding limitations. It was found also that within certain limits and because of the size of the building, a maximum symmetry offered functional and structural advantages. Two geometrical surfaces were selected: the hyperboloid of revolution for the skin and segments of a torus for the plates supporting the floors. The geometries of these two surfaces of revolution were studied to understand their possibilities and limitations.

In designing the building, it was not possible to find a satisfactory approach for many problems. Among them: a continuous floor for the whole building with a minimum slope; a continuous system of circulation; orientation of the people inside of the building in relation with the outside; a means for taking full advantage of the characteristics of both surfaces of revolution; a means to integrate the roof with the rest of the building as a continuous element.

The accompanying drawings and models illustrate three schemes: **VINO, PAN, QUESO.**

NOTES ON TWO SURFACES OF REVOLUTION

TORUS

(1) The "classical" Torus is a continuous double-curved surface generated by revolving a circle or an ellipse around a coplanar axis which does not contain the center of the circle or ellipse.

CIRCULAR OPEN RING TORUS

(2) The surface considered in this paper belongs to the group of Circular Open Ring Tori.

(3) A circular torus is determined by two radii (Fig. 1)
R_2: radius of the generating circle.
R_1: radius of rotation; that is the distance from the center of the circle to the (Z) axis of rotation.
Different sets of values R_2 and R_1 determine a circular torus.
When:

$R_2 > 0;\ R_1 > R_2$	Open Ring Torus
$R_2 > 0;\ R_1 > 0;\ R_1 < R_2$	Closed Torus

Other sets of values will determine respectively:

$R_2 > 0;\ R_1 = 0$	Sphere
$R_2 = 0;\ R_1 > 0$	Circle
$R_2 = 0;\ R_1 = 0$	Point

(4) Certain tori are special cases of the Dupin Cyclidal surfaces. A circular open torus can also be generated as follows: a *Steiner Chain* may be integrated in a series of spheres to form a Dupin Cyclide, and this cyclide must be rotated one phase (α) to generate a torus (Fig. 2)

(5) An open torus can also be generated as follows: a cylinder may be curved to meet itself in such a way that the central axis forms a circle. (Fig. 3)

(6) The surface of the torus is doubly curved with both positive and negative Gaussian curvature.

(7) The surface of the torus is the only surface which can be divided into seven mutually adjacent parts.

(8) The torus has two lines of constant curvature which are circles. These lines are also connective lines or canonical curves.

(9) Topologically, the torus, a two-sided surface, has a connectivity of +3 and can be deformed into a *Klein bottle* of one surface.

(10) Surface area of circular torus:

$$S = 4\pi^2 R_1 R_2$$

(11) Volume of circular torus:

$$V = 2\pi^2 R_1 R_2^2$$

(12) Equations of the surface of the torus for:

Cartesian Coordinates.

$$(X^2 + Y^2 + Z^2 + R_1^2 - R_2^2)^2 = 4 R_1^2 (X^2 + Y^2)$$

Cylindrical Coordinates.

$$X = R\cos\Theta;\quad Y = R\sin\Theta;\quad Z = Z$$

$$(\sqrt{R^2\cos^2\Theta + R^2\sin^2\Theta} - R_1)^2 + Z^2 = R_2^2$$

$$(R\sqrt{\cos^2\Theta + \sin^2\Theta} - R_1)^2 + Z^2 = R_2^2$$

$$(R\sqrt{1} - R_1)^2 + Z^2 = R_2^2$$

$$(R - R_1)^2 + Z^2 = R_2^2$$

Spherical Coordinates

$$X = P\sin\phi\cos\Theta;\quad Y = P\sin\phi\sin\Theta;\quad Z = \cos\phi$$

$$(\sqrt{X^2 + Y^2} - R_1)^2 + Z^2 = R_2^2$$

$$(\sqrt{P^2\sin^2\phi\cos^2\Theta + P^2\sin^2\phi\sin^2\Theta} - R_1)^2 + P^2\cos^2\phi = R_2^2$$

$$(P\sin\phi\sqrt{\sin^2\Theta + \cos^2\Theta} - R_1)^2 + P^2\cos^2\phi = R_2^2$$

$$(P\sin\phi\sqrt{1} - R_1)^2 + P^2\cos^2\phi = R_2^2$$

$$(P\sin\phi - R_1)^2 + P^2\cos^2\phi = R_2^2$$

$$P^2\sin^2\phi + R_1^2 - 2P\sin\phi R_1 + P^2\cos^2\phi = R_2^2$$

$$P^2(\sin^2\phi + \cos^2\phi) + R_1^2 - 2P\sin\phi R_1 = R_2^2$$

$$P^2 + R_1^2 - R_2^2 = 2P\sin\phi R_1$$

HYPERBOLOID OF ONE SHEET

(13) The Hyperboloid of one sheet is the locus of the equation:

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} - \frac{Z^2}{c^2} = 1,$$

in three dimentional Cartesian coordinates. It is called a Quadric Surface because it is the locus of a second degree equation.

(14) The surface is doubly ruled. Each point of the surface is in more than one straight line of the surface.

HYPERBOLOID OF REVOLUTION OF ONE SHEET

(15) The hyperboloids of one sheet considered in this paper belong to a group called one sheeted hyperboloids of revolution, where **a = b** and the equation (13) becomes:

$$\frac{X^2 + Y^2}{a^2} - \frac{Z^2}{c^2} = 1$$

The surface becomes a surface of revolution, all sections (XY) perpendicular to the (Z) axis of revolution being circles. The minimum circle at Z = 0 is called *gorge*.

(16) The hyperboloid of revolution of one sheet is the locus described by a hyperbola revolving around its conjugate axis. Hence, all the sections through this axis are hyperbolas. (Fig. 39)

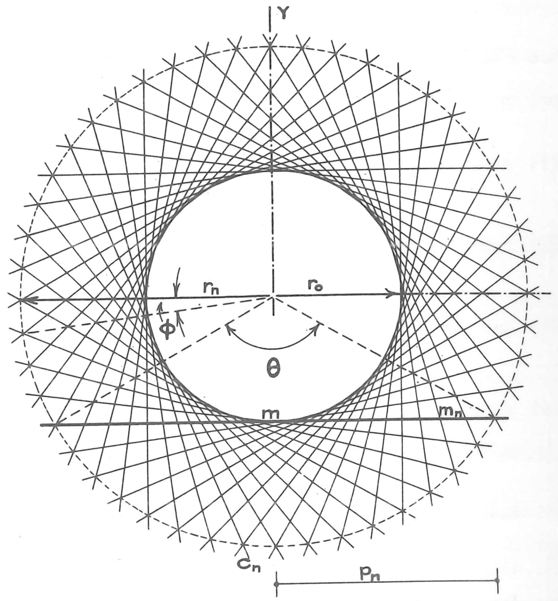
(17) It is also the locus described by a straight generatrix revolving around a non-parallel non coplanar axis (Fig. 40) Furthermore, the projection of this line on a parallel coaxial plane is one of the asymptotes of the hyperbolic sections contained on the plane. Or, as Wren's Theorem States: The section of a one-sheeted hyperboloid of revolution in a plane through the asymptote of a generating hyperbola perpendicular to the plane of this curve is two lines parallel to this asymptote.

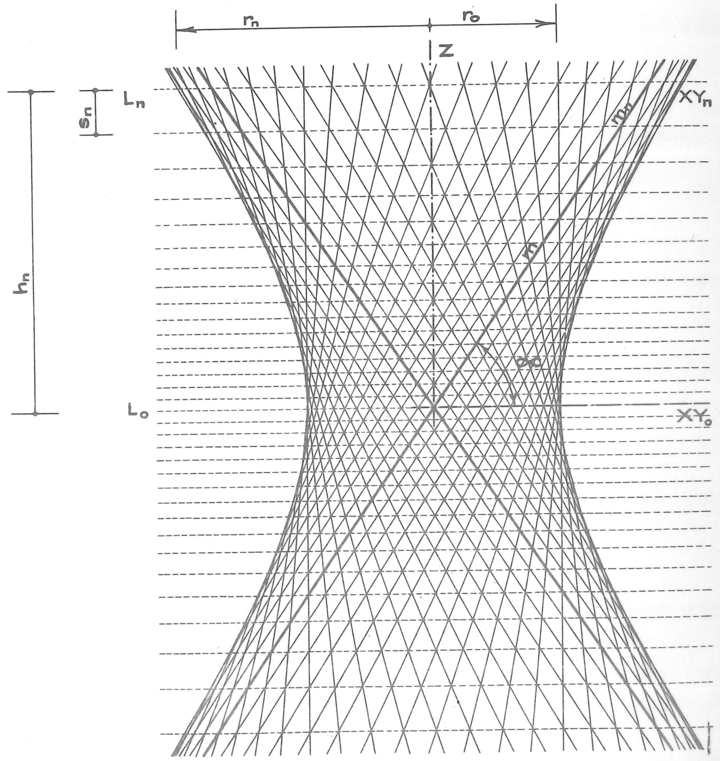
(18) **There are two generatrices that go through each point on the gorge circle.**

Notation (See Figs. 39, 40, 65, 66)

(19) The notation used below refers particularly to the generatrix frozen at equal intervals, or: $\frac{360^\circ}{N} = \phi$

m	:	Straight generatrix or asymptote.
XY	:	Circular sections parallel to XY plane.
XY_o	:	Circular section or gorge circle when Z = 0 (level 0)
XY_n	:	Circular sections at the intersection of a pair of generatrices at level **n.**
r_n	:	Radius of circular sections at level **n.**
r_o	:	Radius of gorge circle—Level 0
m_n	:	A segment of generatrix between consecutive intersections.
h_n	:	Level or distance between gorge level and an intersection at level **n.**
c_n	:	Chord of an arch of a circle between consecutive intersections at any level **n.**
s_n	:	Distance between consecutive intersections levels
L_n	:	Number of intersections levels from plane XY.
p_n	:	Horizontal projection of a segment of generatrix from gorge to an intersection at level **n.**
N	:	Number of intersections at the same level.
ϕ	:	Horizontal projection of the central angle between two consecutive intersections at the same level.
α	:	Angle between a generatrix and XY plane. Called orientation angle.
Θ	:	Central angle of the common trace of a pair of generatrices at level **n.** Called angle of rotation.
ρ	:	Angle at the intersection of a pair of generatrices.





Algebraic Expressions

(See Figs. 39, 40 and 44, 45, 46)

(20) $\Theta = L_n \phi$

(21) $\phi = \dfrac{360^\circ}{N}$

(22) $\tan \alpha = \dfrac{h_n}{p_n} \therefore h_n = p_n \tan \alpha$

(23) $\tan \dfrac{\Theta}{2} = \dfrac{p_n}{r_o} \therefore p_n = r_o \tan \dfrac{\Theta}{2}$

(24) From (22, 23) : $h_n = r_o \tan \dfrac{\Theta}{2} \tan \alpha$

in "parallel" hyperbolas A, B, C, . . . etc. : $h_nA = h_nB = h_nC$ . . . and $\tan \dfrac{\Theta}{2}$ is a constant k, therefore:

$(r_{oA} \tan \alpha_A)k = (r_{oB} \tan \alpha_B)k = (r_{oC} \tan \alpha_C)k$ . . . etc.

$r_{oA} \tan \alpha_A = r_{oB} \tan \alpha_B = r_{oC} \tan \alpha_C$ . . . etc.

(25) $r_{oB} = \dfrac{r_{oA} \tan \alpha_A}{\tan \alpha_B}$, etc.

(26) $\tan \alpha_B = \dfrac{r_{oA} \tan \alpha_A}{r_{oB}}$, etc.

(27) $\left(\dfrac{m}{2}\right)^2 = h_n^2 + p_n^2 \therefore m = 2\sqrt{h_n^2 + p_n^2}$

(28) From (23, 27) : $m = 2\sqrt{h_n^2 + \left(r_o \tan \dfrac{\Theta}{2}\right)^2}$

(29) From (24) : $\tan \alpha = \dfrac{h_n}{r_o \tan \dfrac{\Theta}{2}}$

(30) $\cos \dfrac{\Theta}{2} = \dfrac{r_o}{r_n} \therefore r_n = \dfrac{r_o}{\cos \dfrac{\Theta}{2}}$

(31) $r_n = \sqrt{p_n^2 + r_o^2}$

(32) $\tan (n) \dfrac{\phi}{2} = \dfrac{p_n}{r_o} \therefore p_n = r_o \tan (n) \dfrac{\phi}{2}$

(33) From (31, 32) : $r_n = \sqrt{\left[r_o \tan (n) \dfrac{\phi}{2}\right]^2 + r_o^2}$

(34) From (22, 32) : $h_n = r_o \tan (n) \dfrac{\phi}{2} \tan \alpha$

(35) $S_n = h_n - h_{n-1}$

(36) From (34, 35) :

$$S_n = r_o \tan \alpha \left[\tan (n) \frac{\phi}{2} - \tan (n-1) \frac{\phi}{2}\right]$$

(37) $\sin \alpha = \dfrac{S_n}{m_n} \therefore m_n = \dfrac{S_n}{\sin \alpha}$

(38) From (36, 37) :

$$m_n = \frac{r_o \tan \alpha \left[\tan (n) \frac{\phi}{2} - \tan (n-1) \frac{\phi}{2}\right]}{\sin \alpha}$$

(39) $\sin \dfrac{\phi}{2} = \dfrac{c_n}{2\, r_n} \therefore c_n = 2 r_n \sin \dfrac{\phi}{2}$

(40) From (33, 39) :

$$c_n = 2\sqrt{\left[r_o \tan (n) \frac{\phi}{2}\right]^2 + r_o^2}\ \sin \frac{\phi}{2}$$

(41) $\sin \dfrac{\rho}{2} = \dfrac{c_n}{2 m_n}$

(42) From (40, 41) :

$$\sin \frac{\rho}{2} = \frac{\sqrt{\left[r_o \tan (n) \frac{\phi}{2}\right]^2 + r_o^2}\ \sin \frac{\phi}{2} \sin \alpha}{r_o \tan \alpha \left[\tan (n) \frac{\phi}{2} - \tan (n-1) \frac{\phi}{2}\right]}$$

LIST OF PLATES

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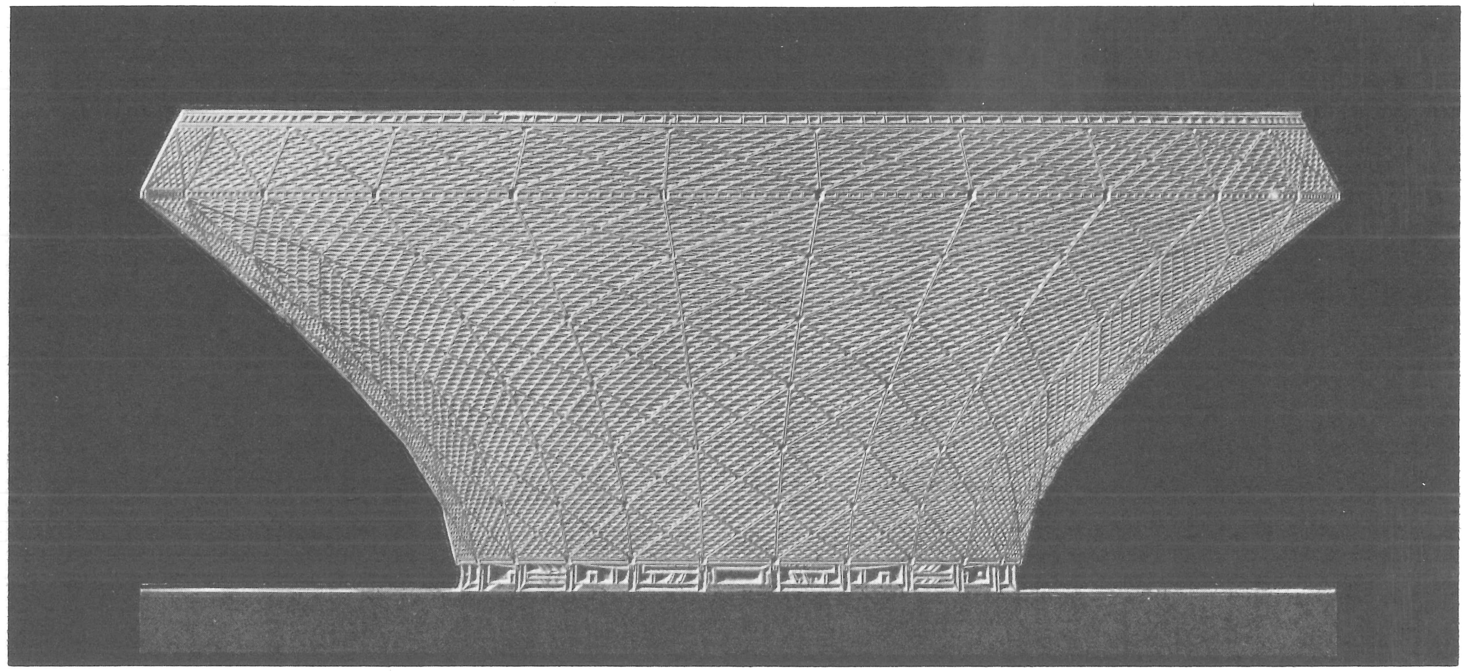
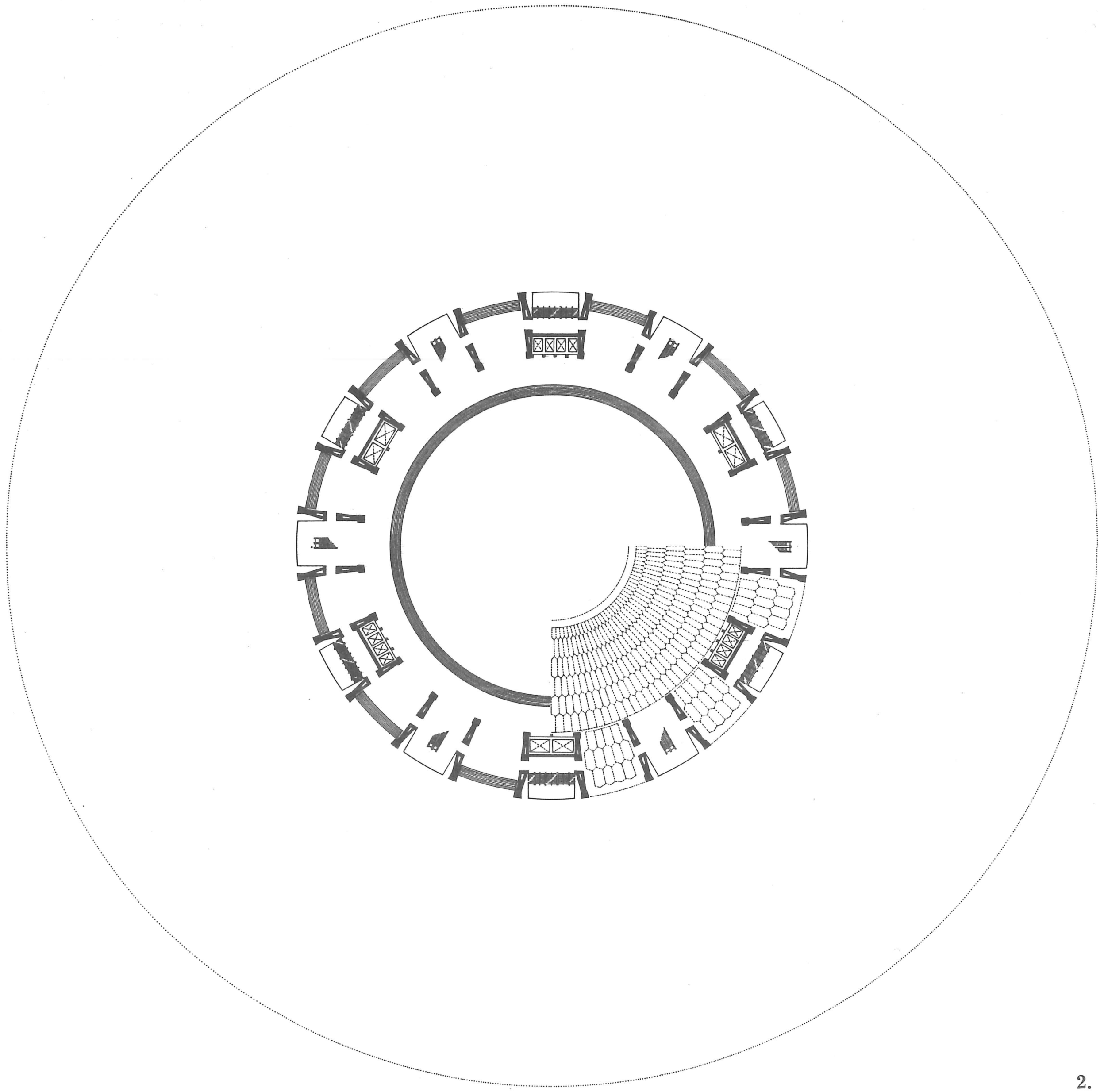


PLATE I. WORLD'S FAIR—SCHEME VINO 1. Elevation

SCHEME VINO It was conceived as a reinforced concrete structure wrapped with a metal tensile skin. The roof is a stretched membrane. Twelve supporting piers on reinforced concrete contain the vertical circulation composed of fire stairs, elevators, escalators, ducts and pipes. Six main floors and a balcony constitute the exhibition space. Below each main floor there are secondary mezzanines with lounges, rest rooms, offices, storage, mechanical equipment, etc. Principal dimensions: Total height: 340 feet. Diameter at ground floor level: 400 feet. Diameter at balcony level (maximum diameter): 860 feet.

BRUEGEL THE ELDER — Fragment of a Print



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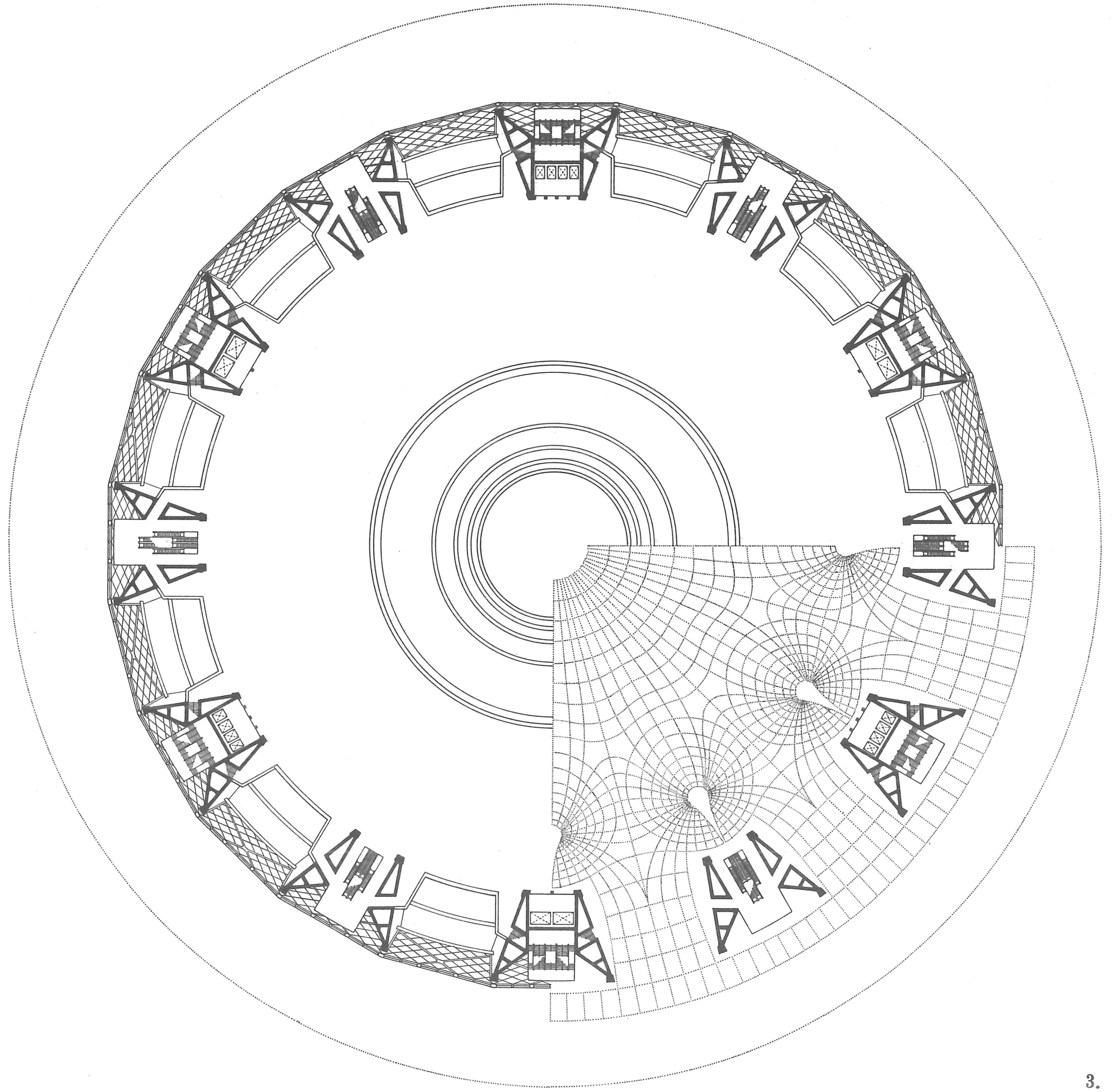

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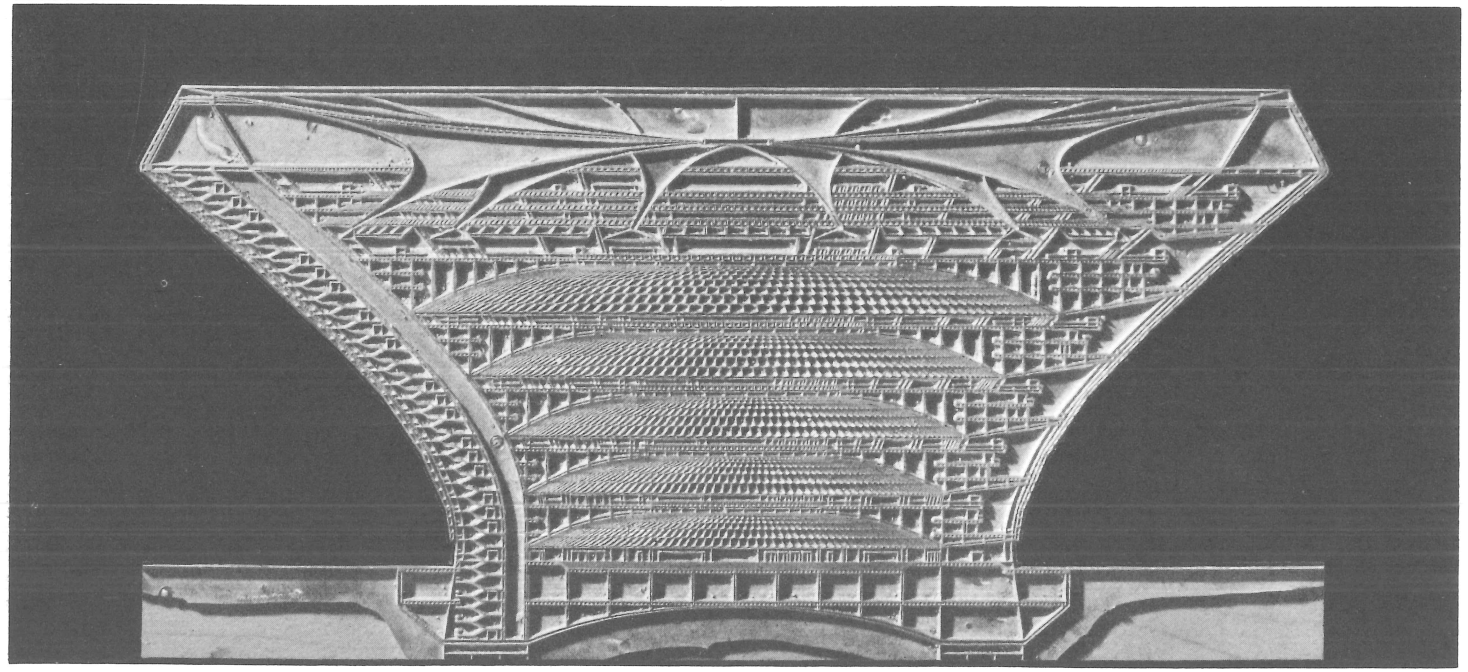
PLATE II. WORLD'S FAIR—SCHEME VINO   2. Plan at ground floor level   3. Plan at upper floor level



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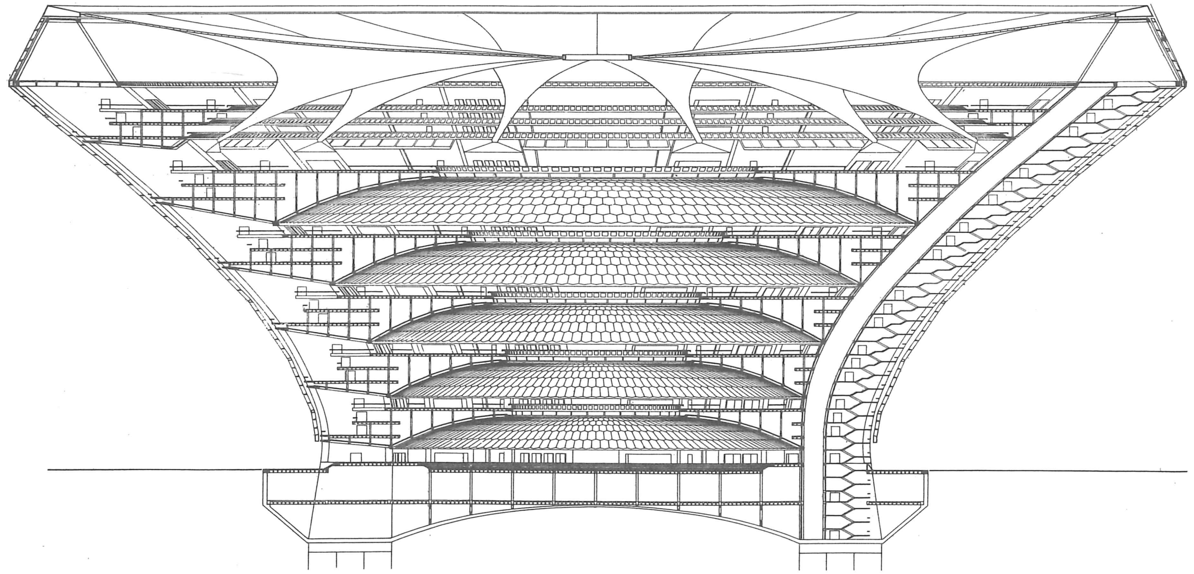

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PLATE III. WORLD'S FAIR—SCHEME VINO 4-5. Sections. Half section through lounges. Half section through vertical cores.

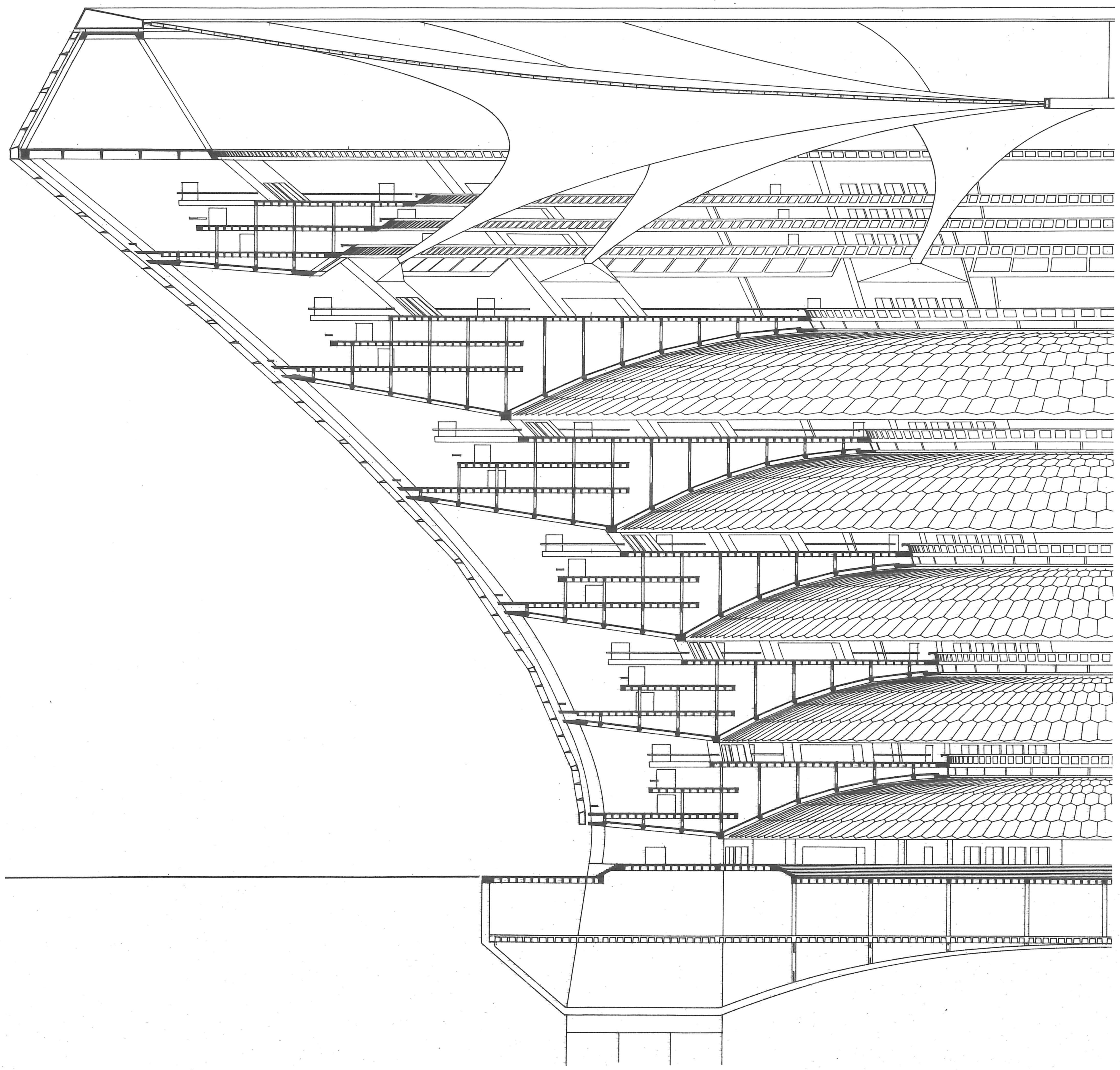
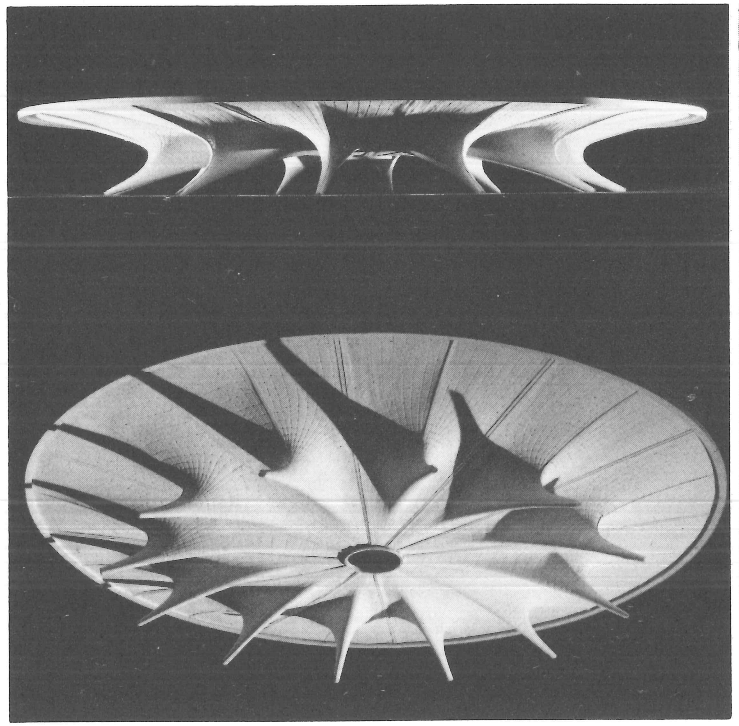
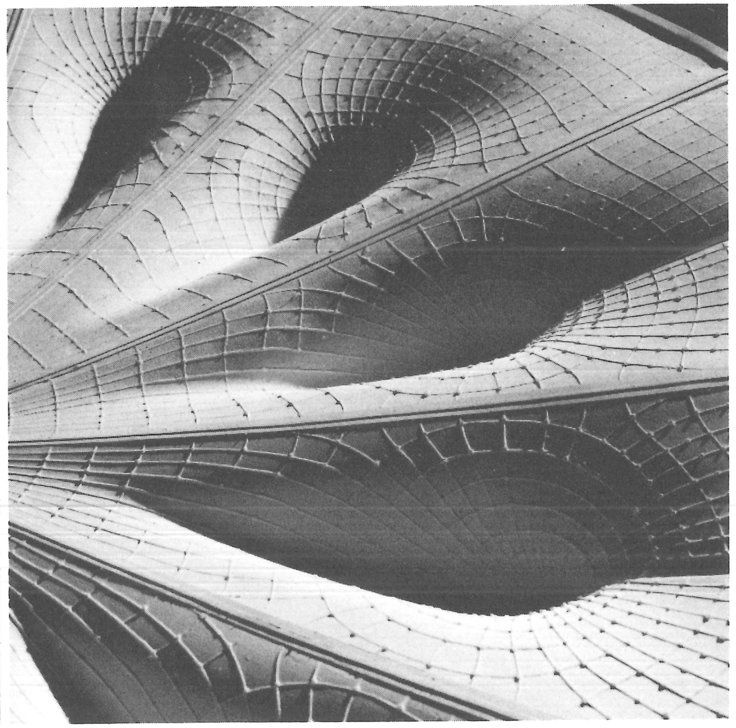


PLATE IV. WORLD'S FAIR—SCHEME VINO 6. Half section through lounges. The exhibitions floors recede on the periphery providing a downward view of the surrounding plaza.





7. 8. 9.

The roof is a variation of the membranal structures developed by the author and presented on the *Students Publications of the School of Design Volume 6, Number 3, May 1957*, under the title: "Two types of membranal structures." **Caminos, Gallo, Guarnieri.** Its shape is essentially determined by a membrane hanging from a circular perimeter. To minimize the flapping or vibrating effect caused by the wind and at the same time to provide drainage of the surface, particular points are pulled down outwards, generating the form. The surface is composed of equal sectors. There are two characteristic vertical radial sections. A section through a lower vertex determines a directrix composed of two downwardly concave lines: one from the center to the vertex, the other from the vertex to the perimeter. A section equidistant from two adjacent lower vertices determines a directrix composed of one downwardly convex line from the center to the perimeter. In this particular case, the membrane hangs from the circular ring and is anchored to the low vertices. Nevertheless, the shape permits other variation; that is: frozen, the shape may become a shell, the points of support being the low vertices. For practical reasons as can be noticed on the photographs, the model was built as a shell.

10. 11.

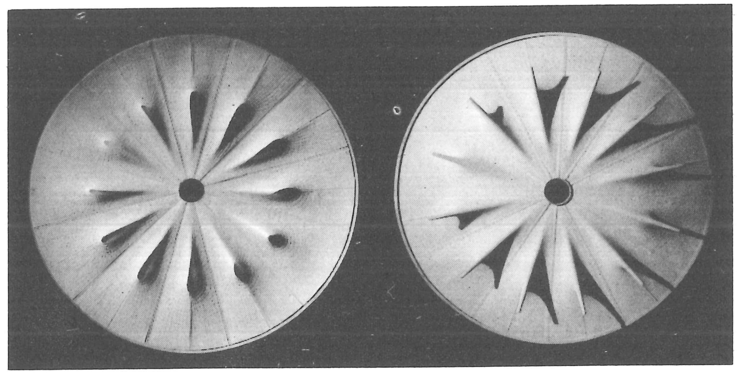


PLATE V. WORLD'S FAIR—SCHEME VINO—MEMBRANAL ROOF
7-8. Elevation and view from beneath. 9. View of the top. 10-11. Top and bottom horizontal projections.

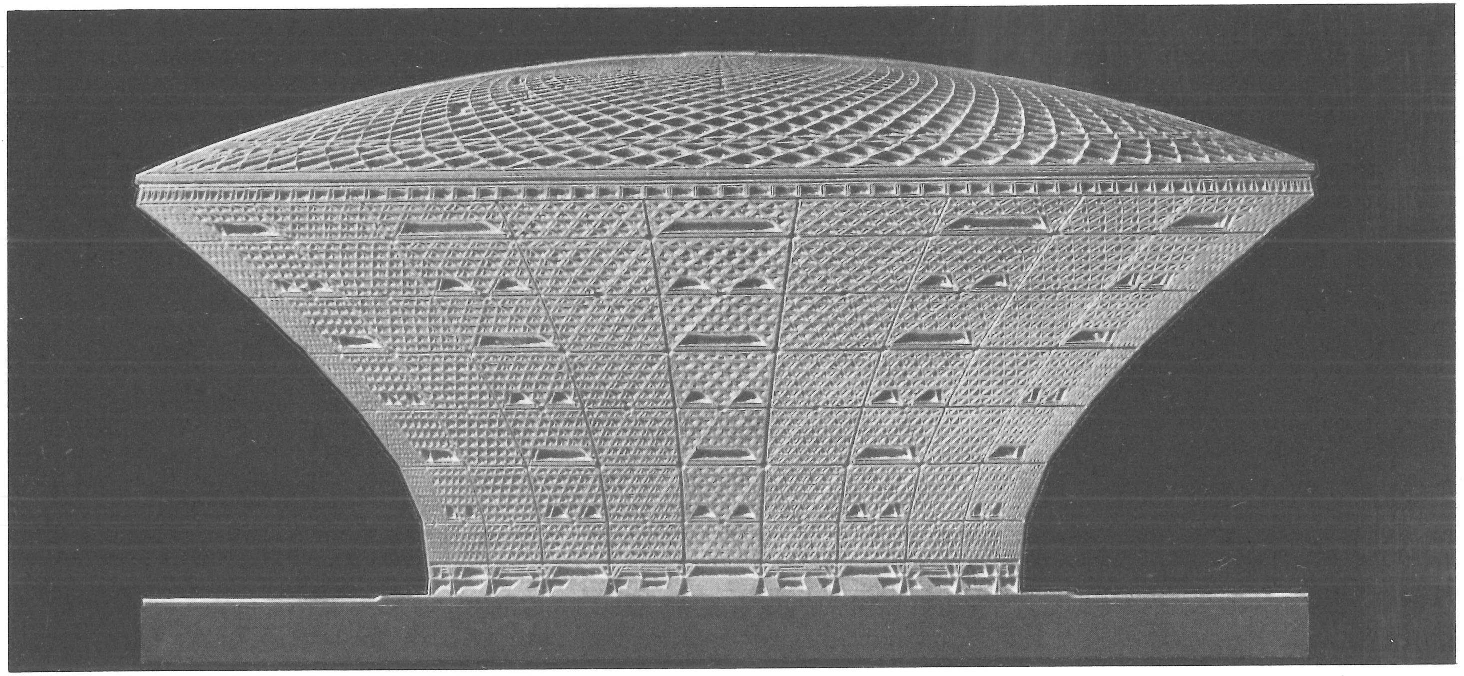
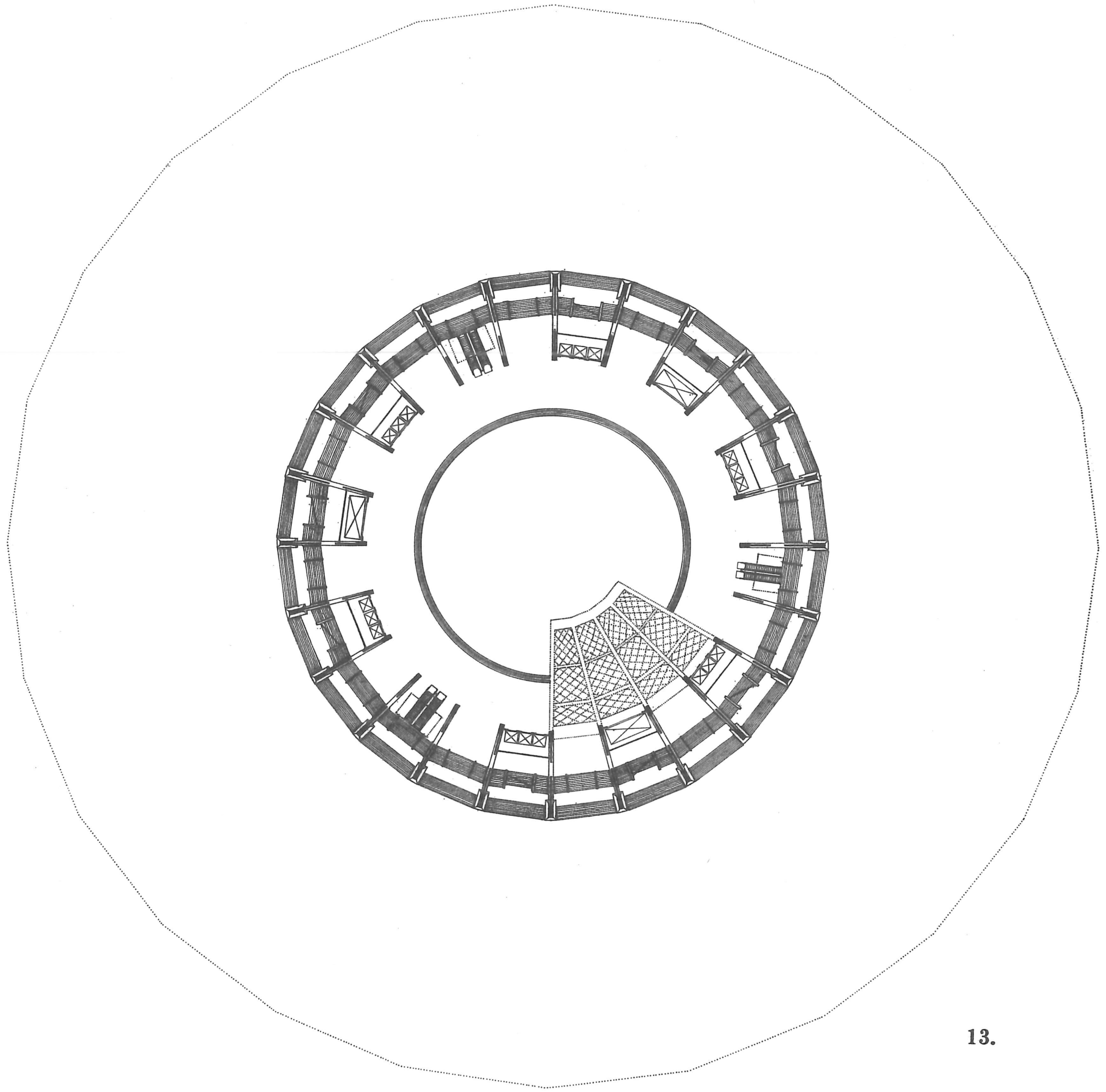


PLATE VI. WORLD'S FAIR—SCHEME PAN 12. Elevation

SCHEME PAN Entire structure in reinforced concrete. The cover is a space frame dome. Twelve supporting piers contain the vertical circulation composed of fire stairs, elevators, escalators, ducts, and pipes. Seven main floors and a balcony constitute the exhibition space. Below each main floor there are secondary mezzanines with lounges, rest rooms, offices, storage, mechanical equipment, etc. As in the other two schemes, expansion joints are in horizontal planes with the purpose of allowing horizontal displacements without losing the advantage of the circular condition. **Principal dimensions: Total height—305 feet. Diameter at ground floor level—375 feet. Diameter at balcony level (maximum diameter)—750 feet.**

BRUEGEL THE ELDER — Fragment of a Print



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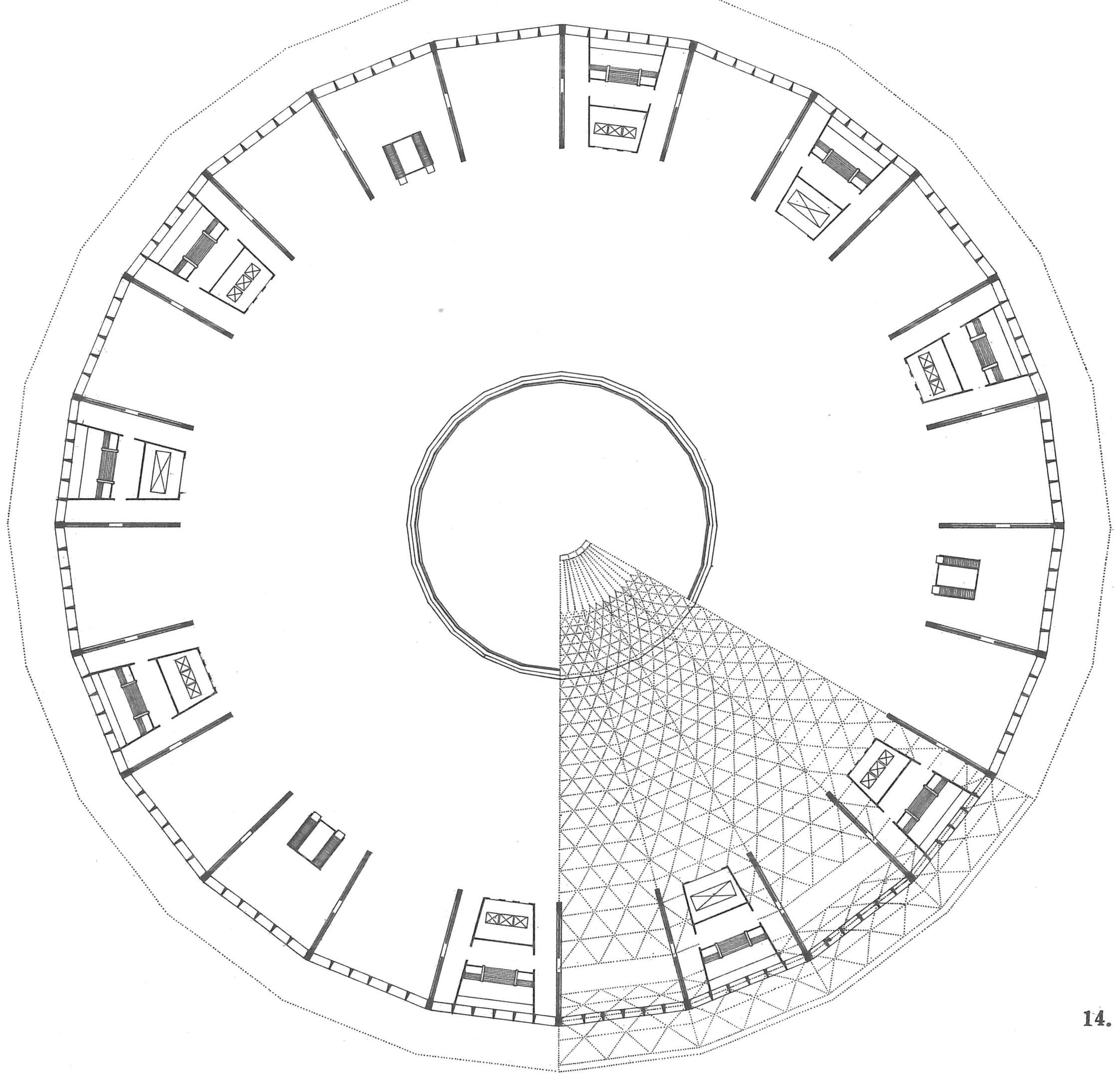
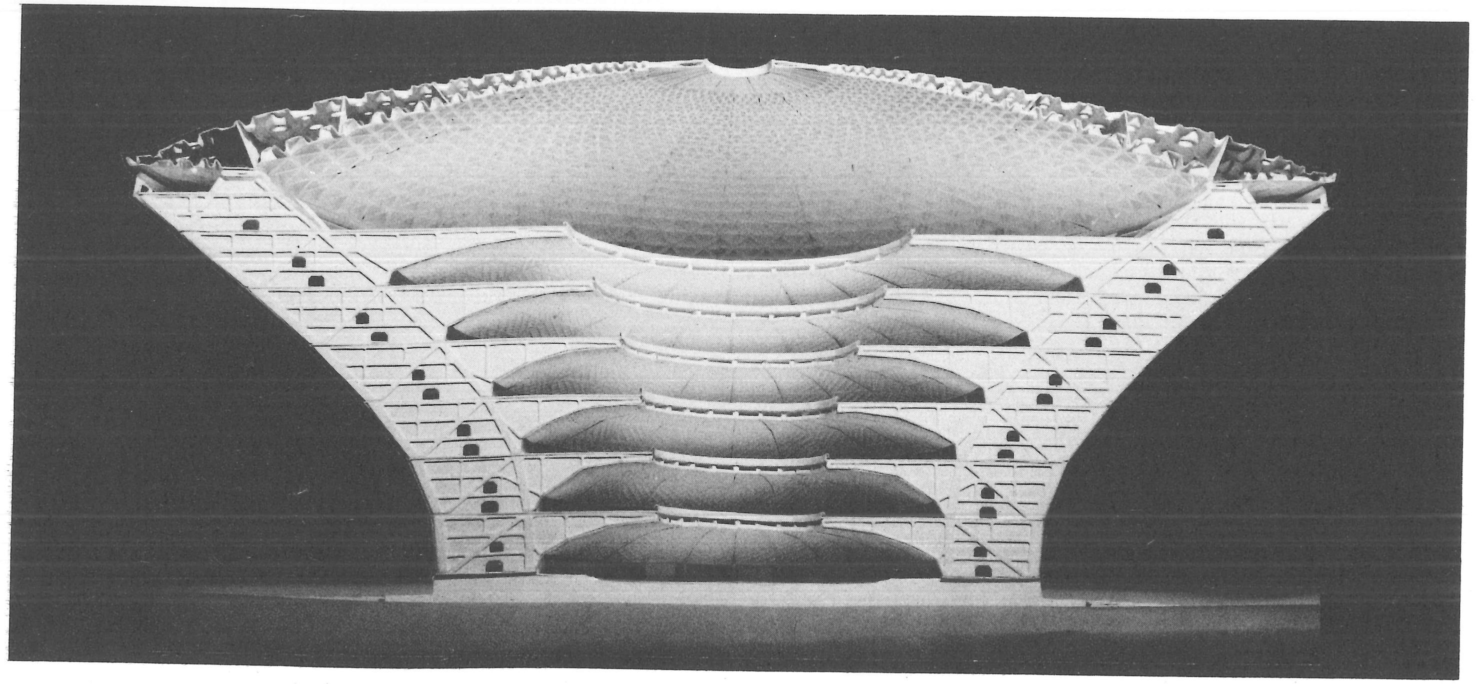


14.

PLATE VII. WORLD'S FAIR—SCHEME PAN 13. Plan at ground floor level. 14. Plan at upper floor level.



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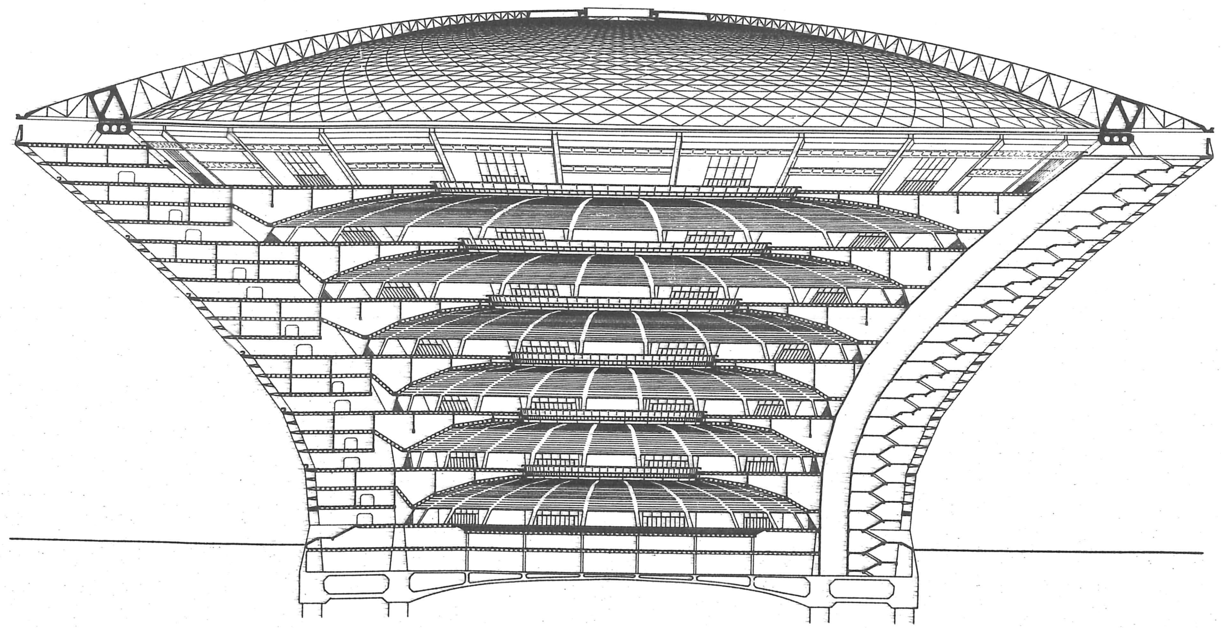


16.

PLATE VIII. WORLD'S FAIR—SCHEME PAN 15-16. Sections

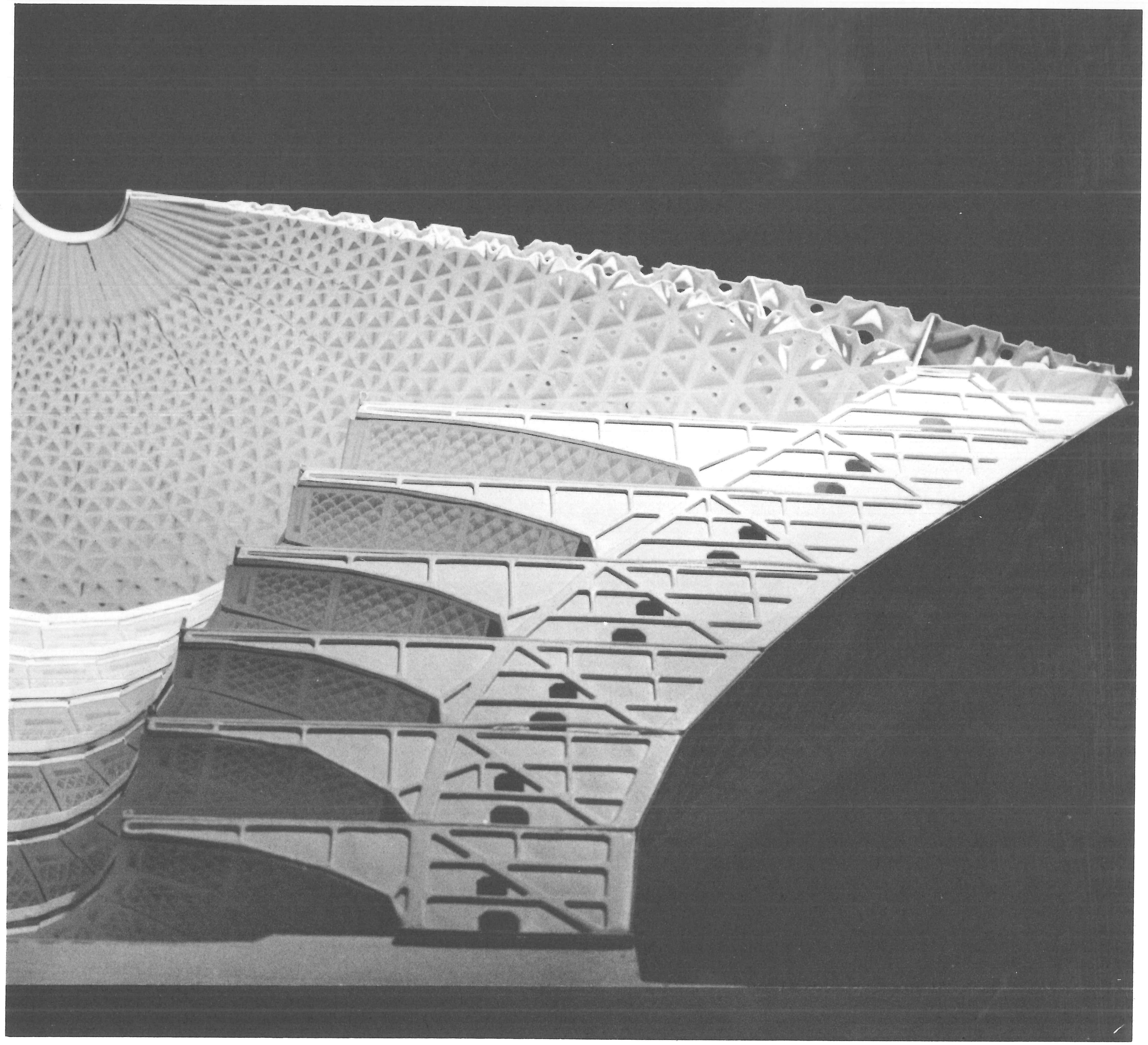


PLATE IX. WORLD'S FAIR—SCHEME PAN 17. Half section (from a model)

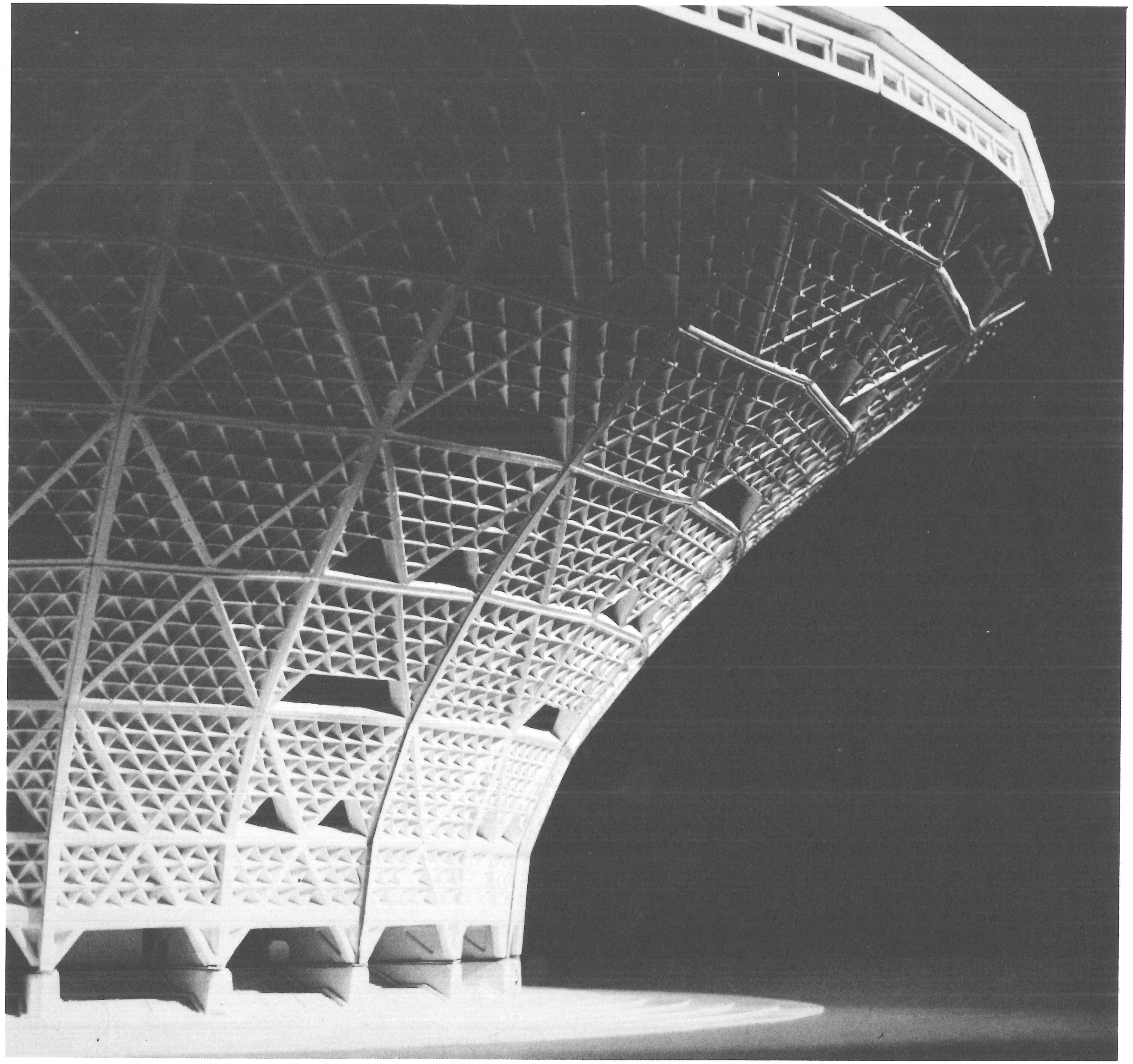


PLATE X. WORLD'S FAIR—SCHEME PAN 18. View of entrances at Plaza level

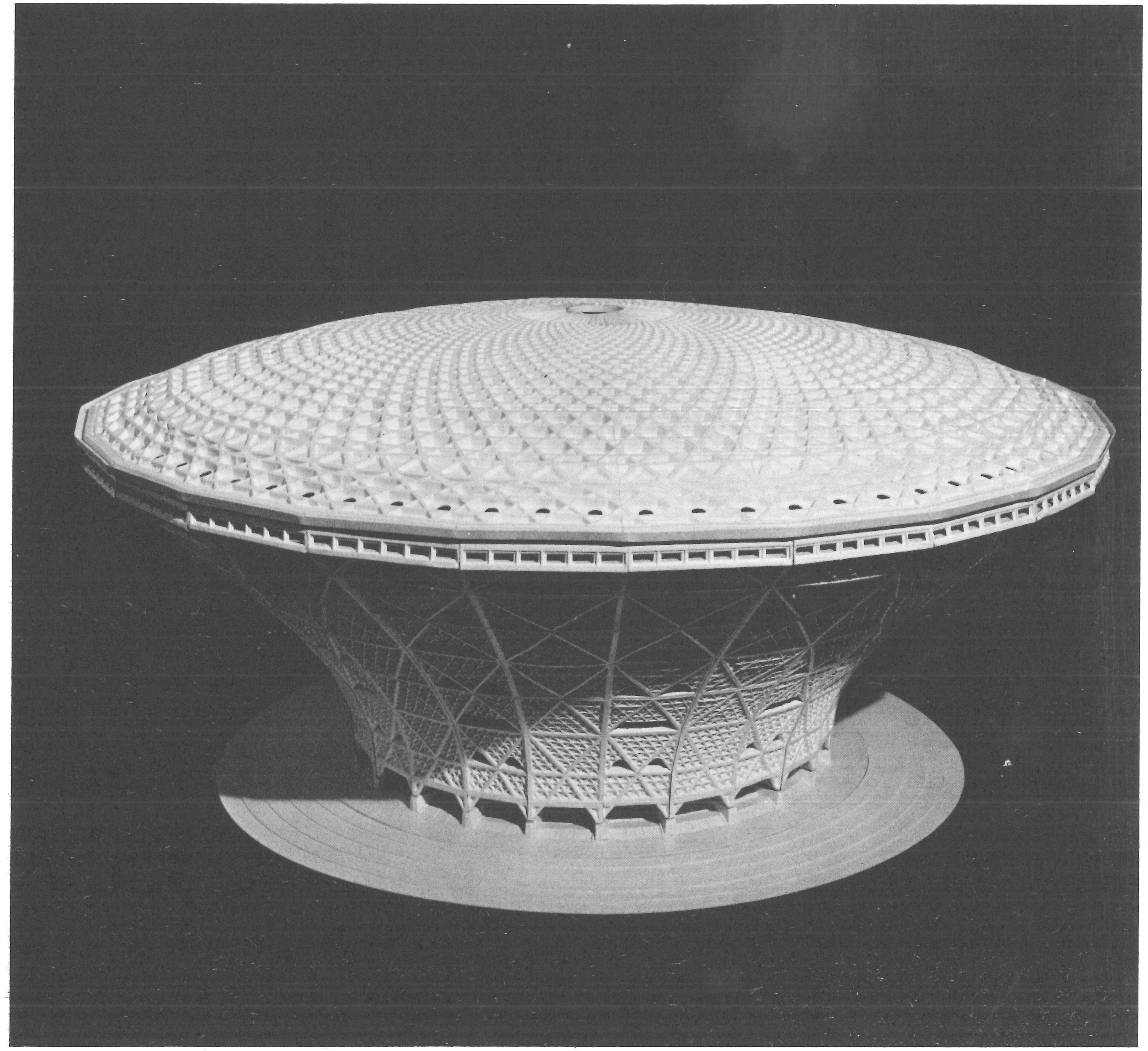


PLATE XI. WORLD'S FAIR—SCHEME PAN 19. Birds eye view of the building.

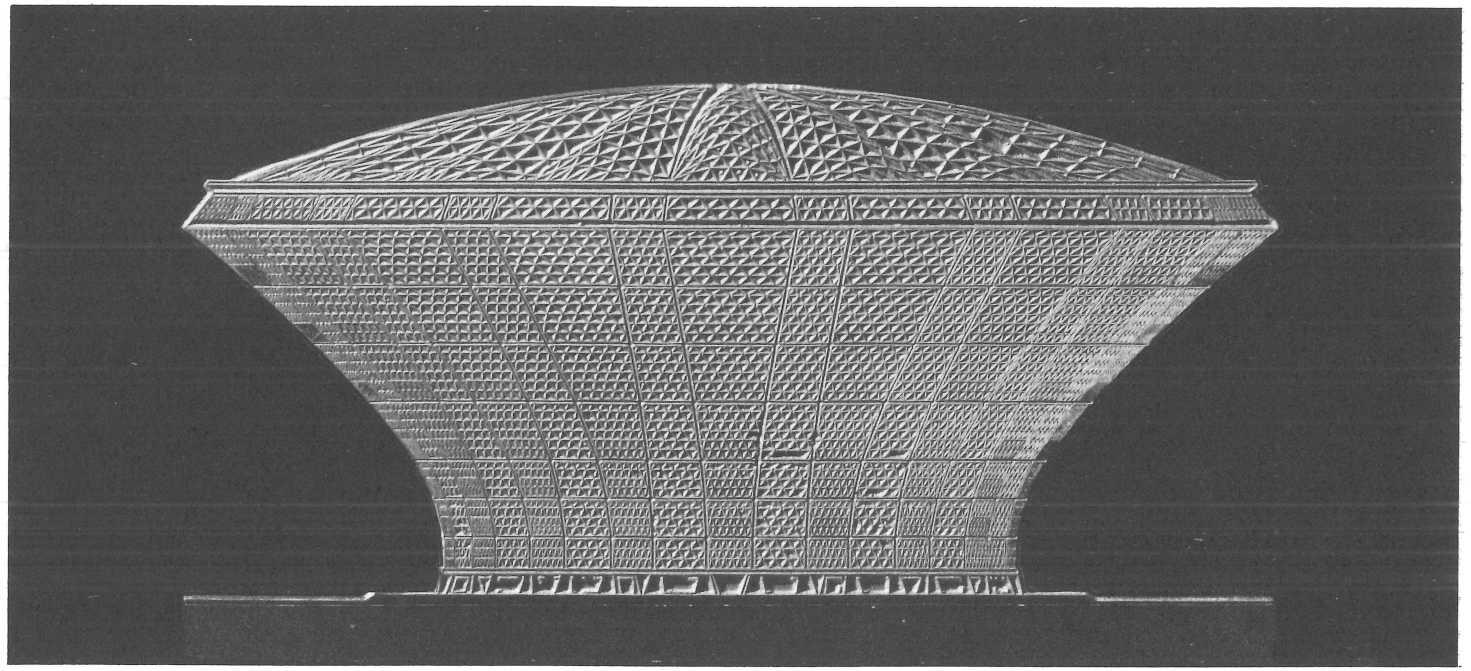
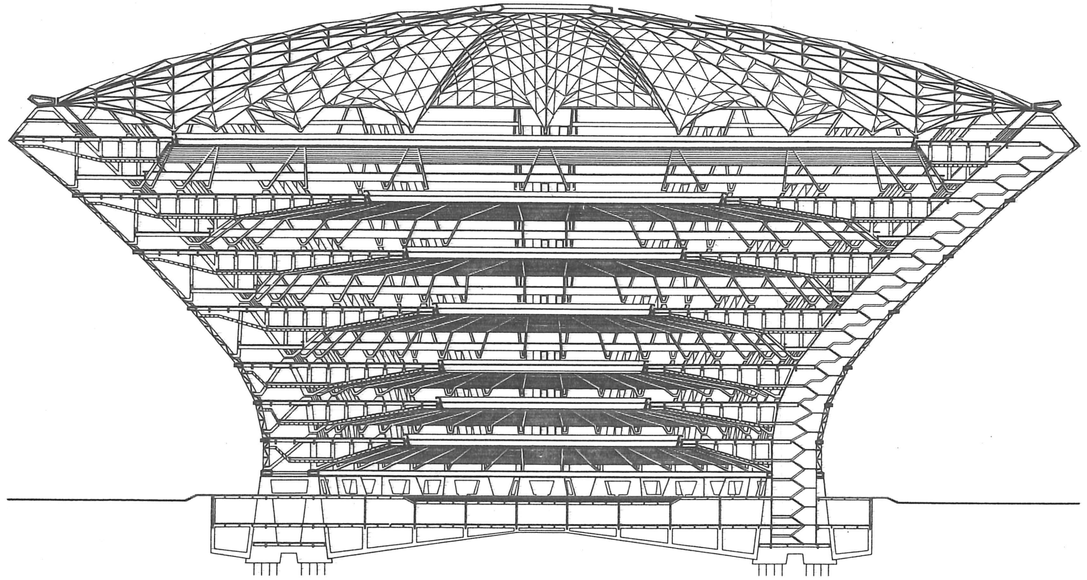


PLATE XII. WORLD'S FAIR—SCHEME QUESO 20. Elevation.

SCHEME QUESO Steel structure. The cover is a double curvature surface derived from a dome. It has 18 low vertices equidistant from the center and equally spaced. Eighteen supporting piers contain the vertical circulation. Seven main floors and a balcony constitute the exhibition space. Below each main floor there are mezzanines with lounges, rest rooms, offices, storage, mechanical equipment, etc. **Principal dimensions: total height—315 feet. Diameter at ground floor level—360 feet. Diameter at balcony level (maximum diameter) 680 feet.**

BRUEGEL THE ELDER — Fragment of a Print



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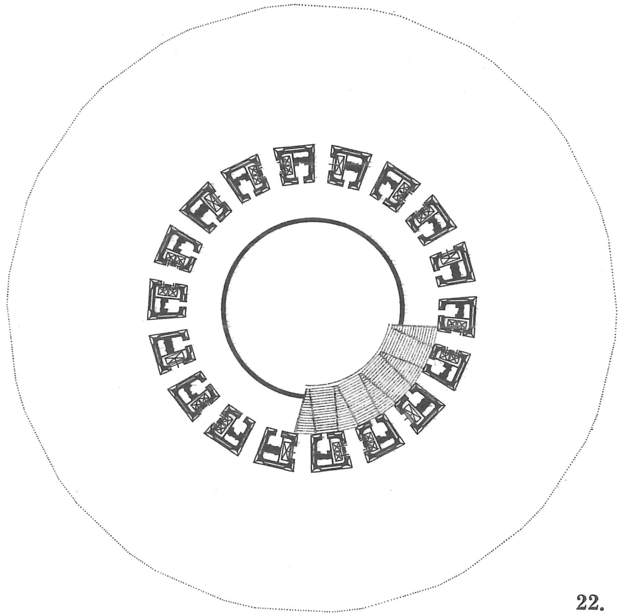

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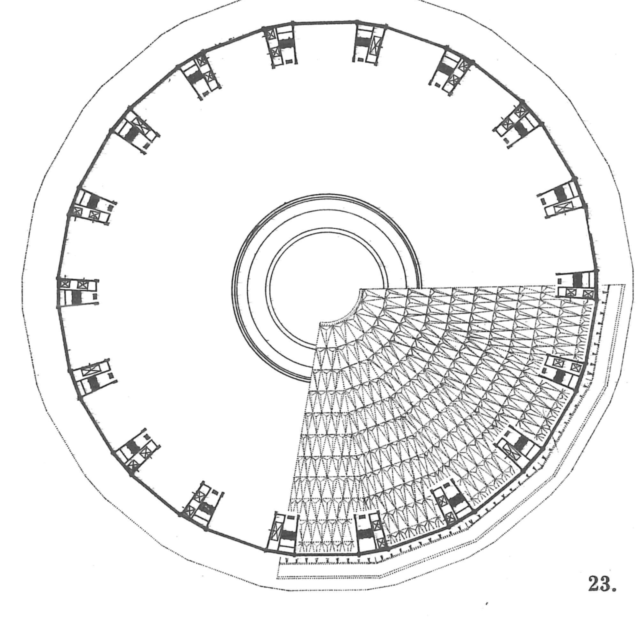
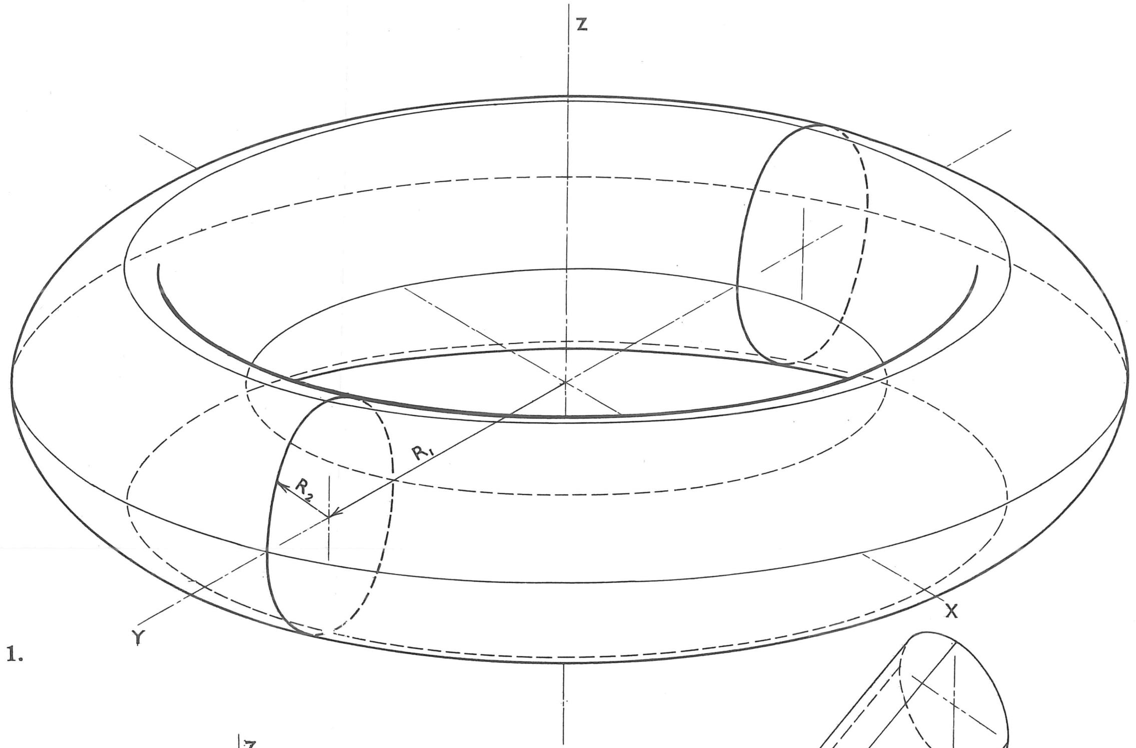


23.

PLATE XIII. WORLD'S FAIR—SCHEME QUESO 21. Section—half section through lounges, half section through vertical cores. 22. Plan at ground floor level. 23. Plan at upper floor level.



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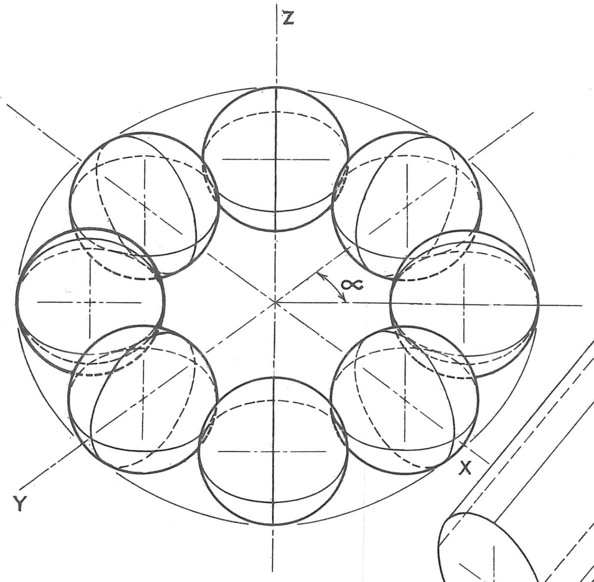

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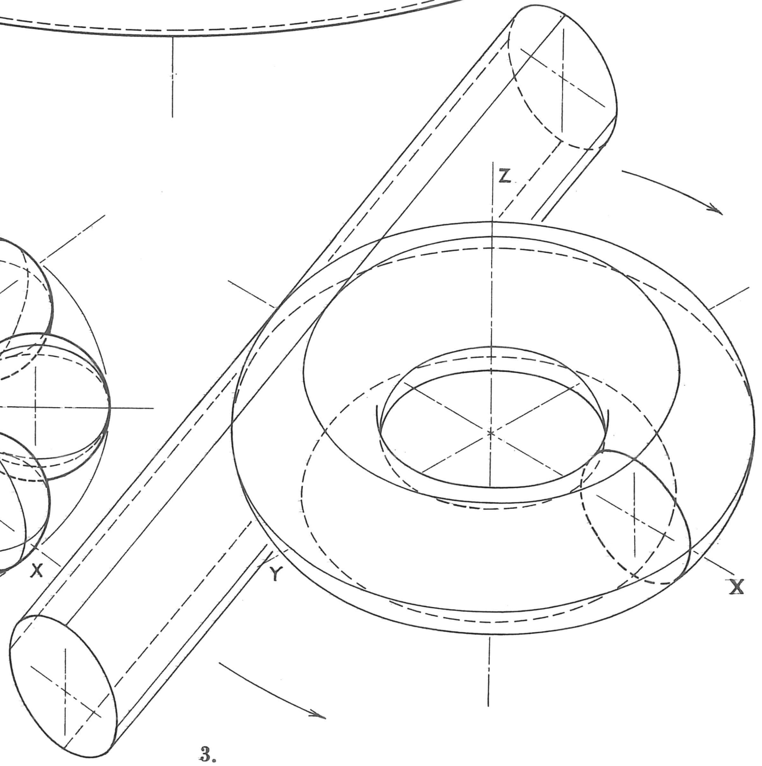
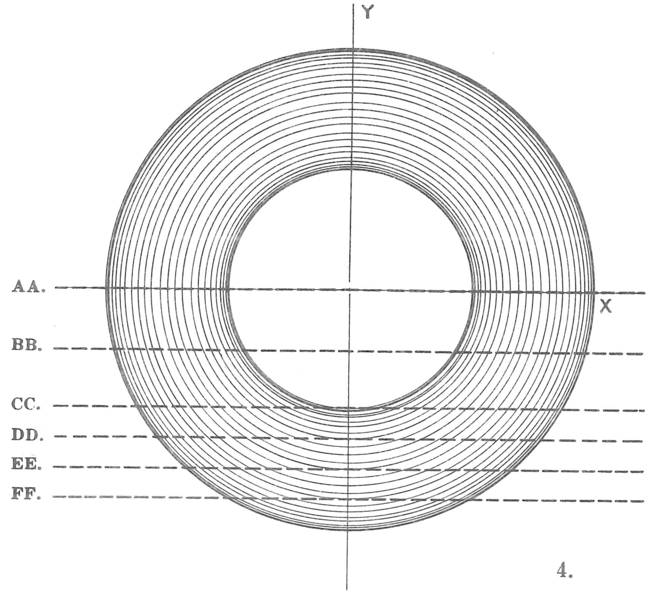


3.

PLATE I. TORUS GENERATION *(Isometric views)* 1. By rotation of a circle. 2. By rotation of a Dupin Cyclide. one phase α. 3. By curving a cylinder.



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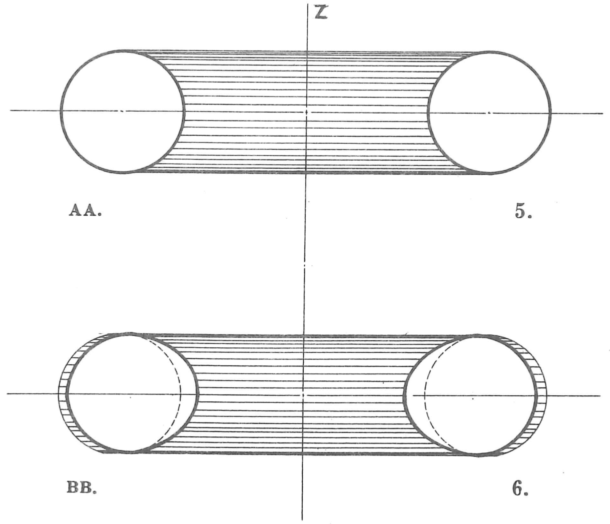


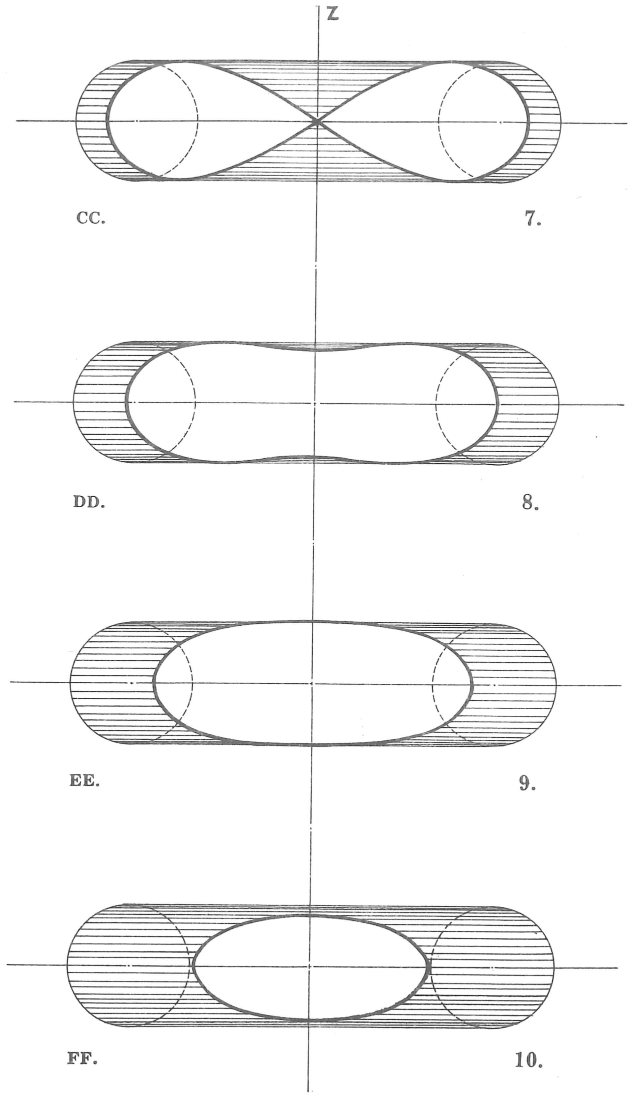


PLATE II. TORUS—PLANE SECTIONS 4. Horizontal projection of torus indicating sections perpendicular to XY plane. These sections are called: Spiric Lines of Perseus. 5. SECTION AA: Circles 6. SECTION BB: Cassinian Curve 7. SECTION CC: (at a distance $R_2 - R_1$ from Z axis) Lemniscate of Bernoulli. 8. SECTION DD: Cassinian Curve. 9. SECTION EE: (at a distance R_2 from Z axis). Elliptical type. 10. SECTION FF: Elliptical type.

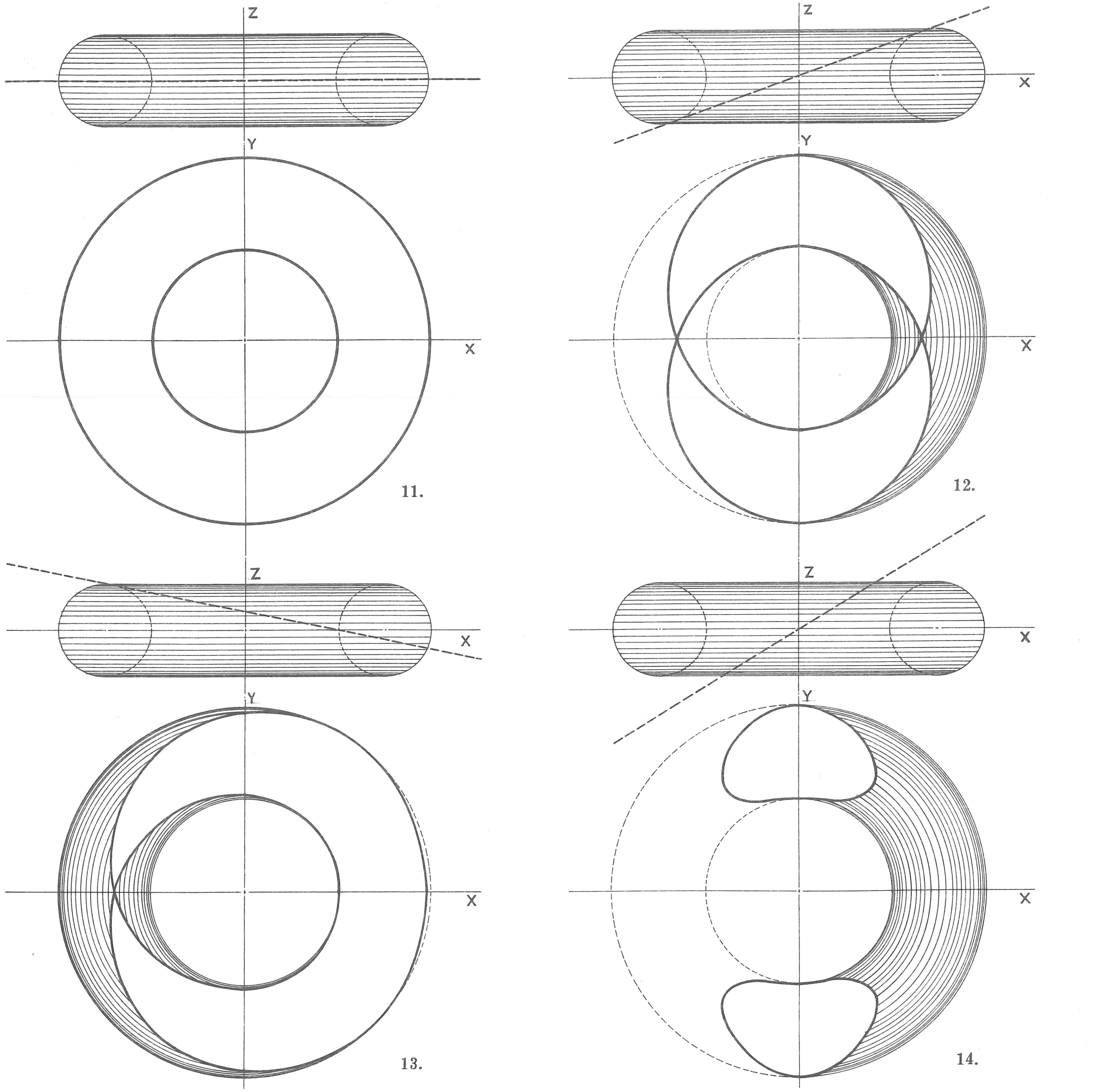


PLATE III. TORUS—PLANE SECTIONS *(Vertical and horizontal projections)* 11. Section through *XY* plane : Concentric circles. 12. Section tangent to two points on surface: Interlocking circles. 13. Section tangent to one point on surface : Epi— and Hypocycloids. 14. Oblique section through center of surface: Cycloidal type.

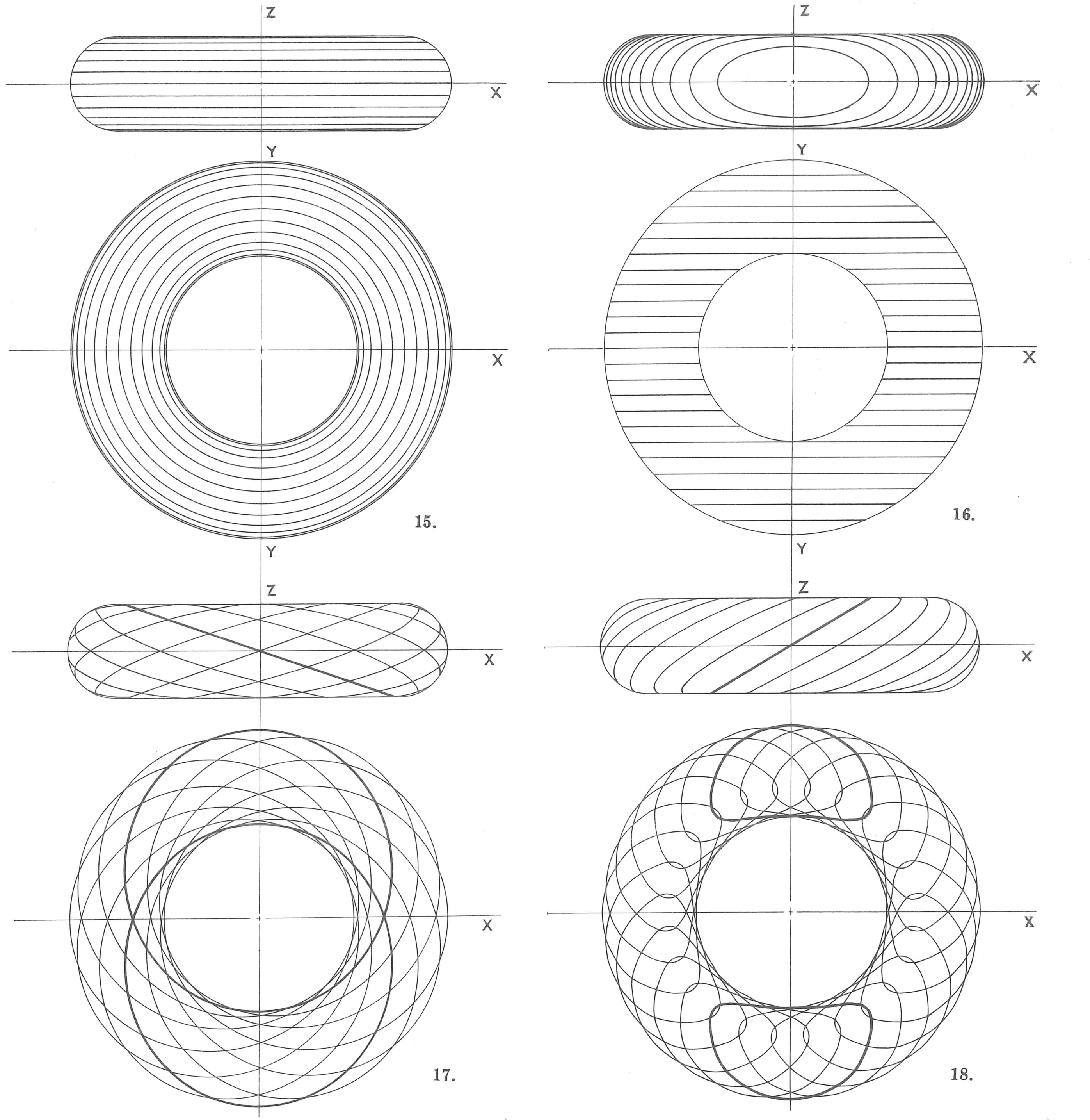
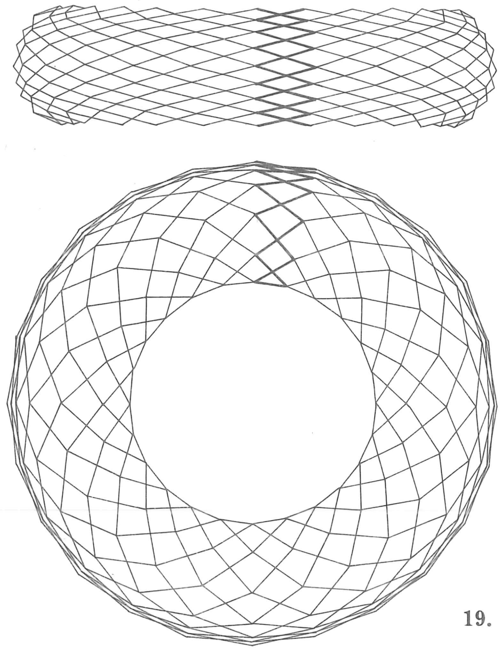


PLATE IV—TORUS—TRANSLATION AND ROTATION OF PLANE SECTIONS *(Vertical and horizontal projections)* 15. Sections perpendicular to *Z* axis: Circles. 16. Sections parallel to *XZ* plane (See sections, Figs. 11, 12, 13, 14) 17. Rotation of interlocking circles. (See section, Fig. 11) 18. Rotation of a Cycloidal type section. (See section, Fig. 13)



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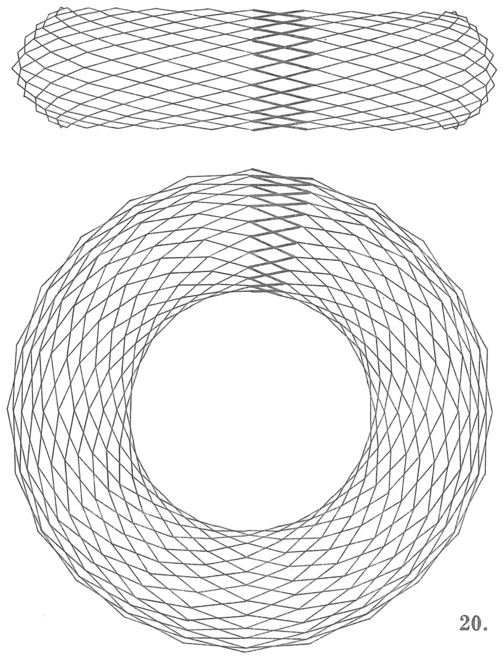

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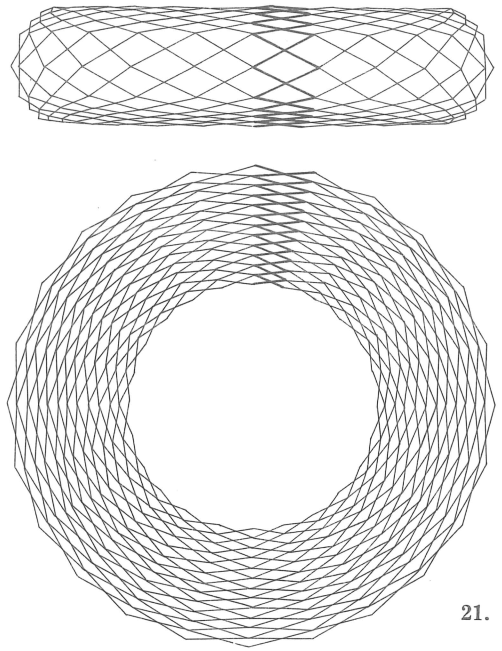

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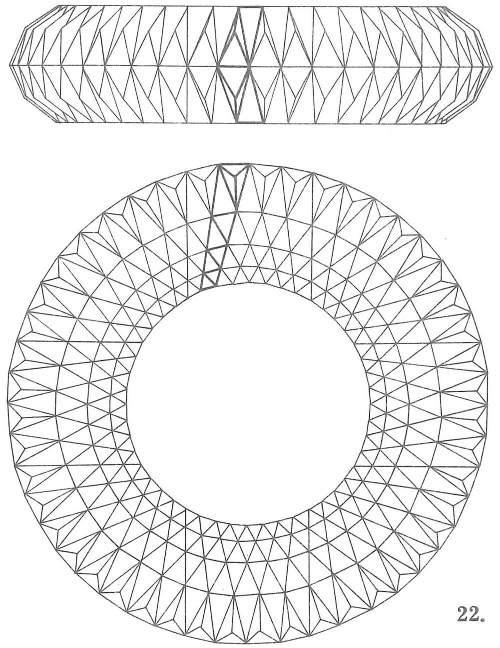
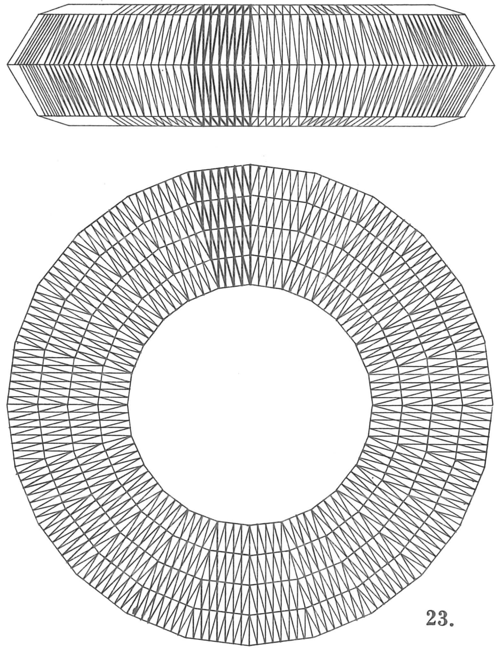


22.

PLATE V. TORUS—DIVISION OF SURFACE WITH CHORDS AND/OR SEGMENTS. *(CHORD: The chord of a circular arch. SEGMENT: the chord of a non-circular arch.)* 19. DIAMONDS. Equal length segments between circular sections. (See PLATE III AA). 20. DIAMONDS. Segments between circular sections equally divided. 21. DIAMONDS. Segments between circular sections equally divided in horizontal projections. 22. TRIANGLES, Chords between Cassinian sections defined by segments. (See PLATE III BB).



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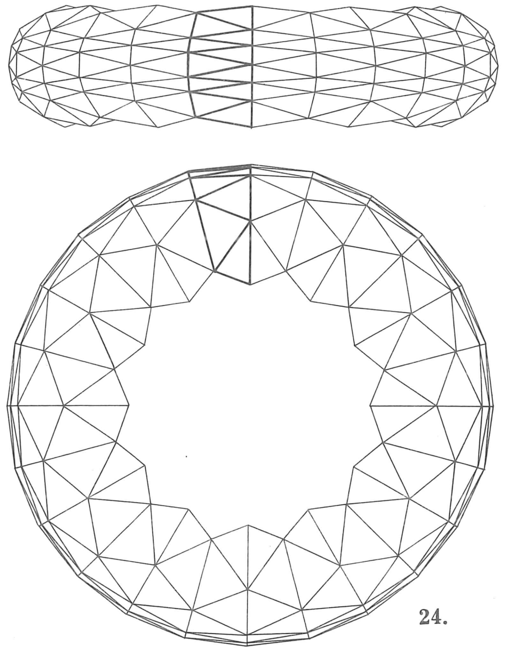

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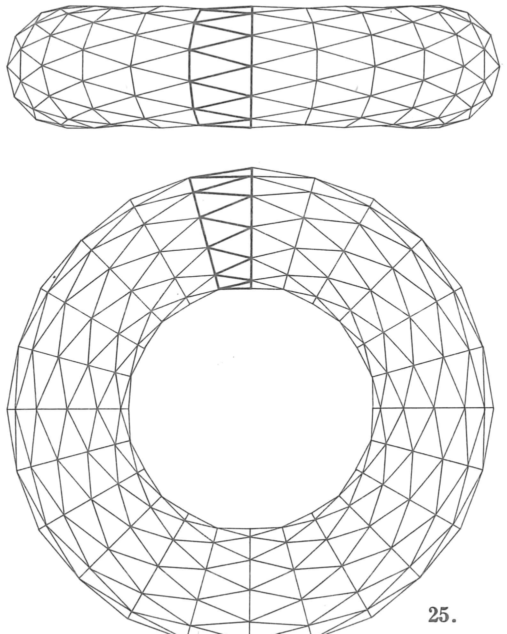

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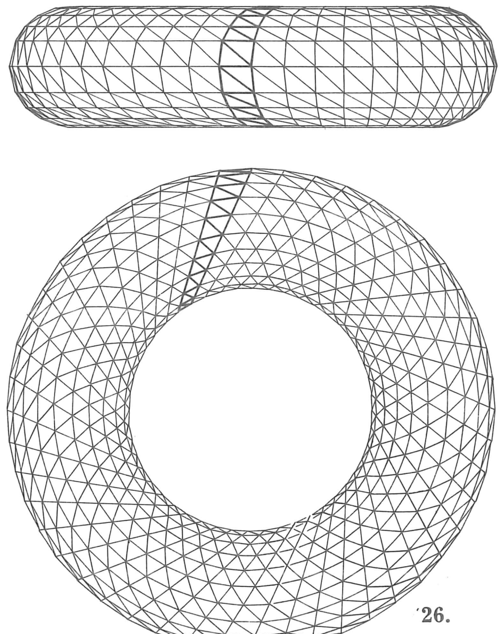
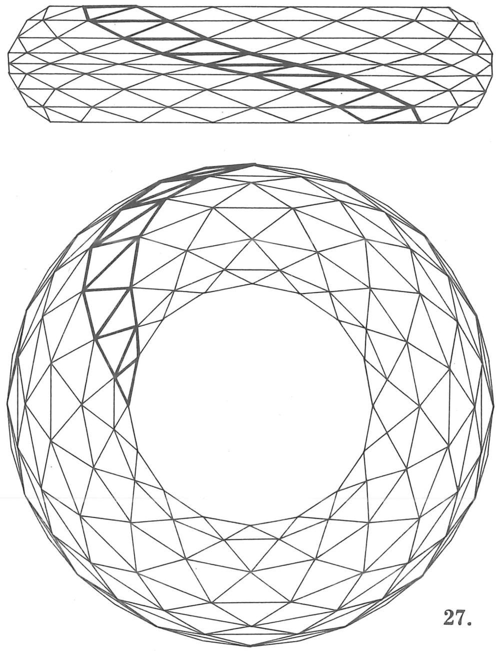


26.

PLATE VI. TORUS—DIVISION OF SURFACE WITH CHORDS AND/OR SEGMENTS. *(CHORD: The chord of a circular arch. SEGMENT: The chord of a non-circular arch.)* 23. TRIANGLES. Basic division: circular sections equally divided in horizontal projection. Subdivisions: chords equally divided. 24. TRIANGLES. Similar Fig. 18. Equal length segments between circular sections. Chords of different lengths. 25. TRIANGLES. Similar Fig. 19. Segments between circular sections equally divided. 26. TRIANGLES. Chords and segments between half cassinian sections divided in equal segments.



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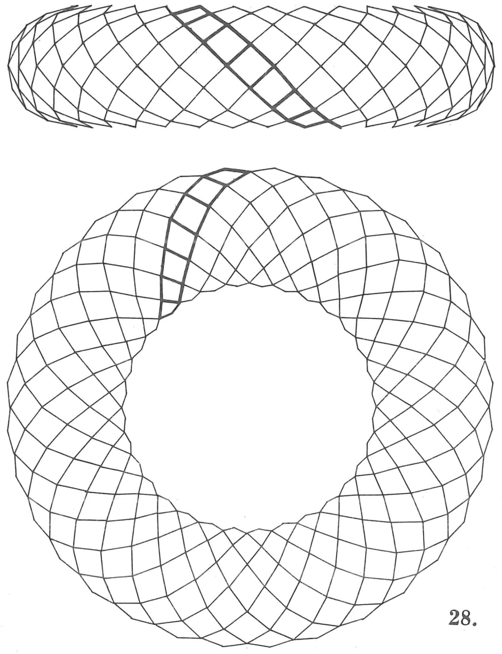

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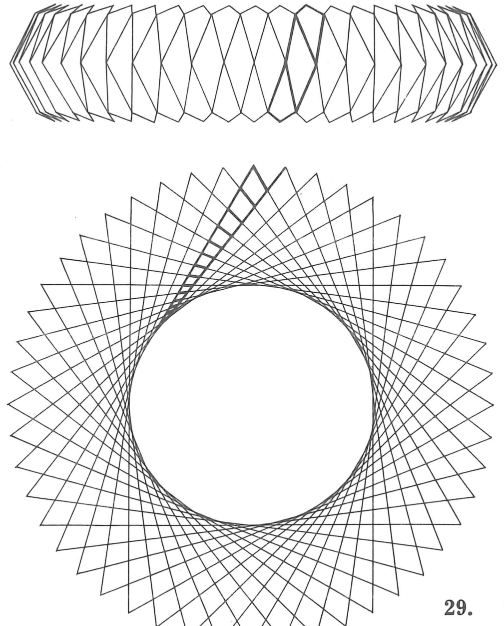

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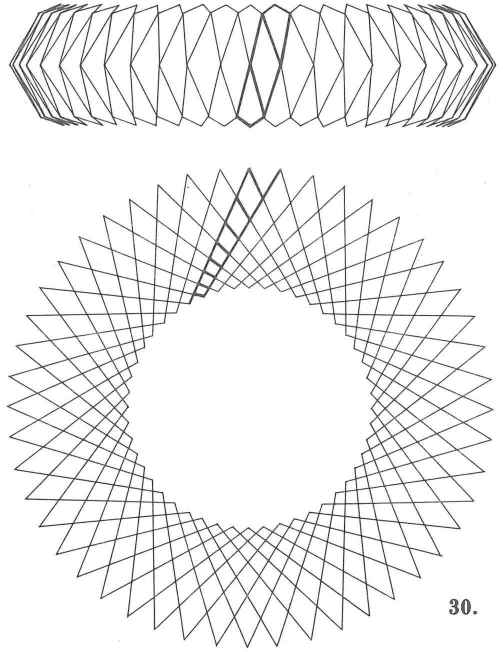
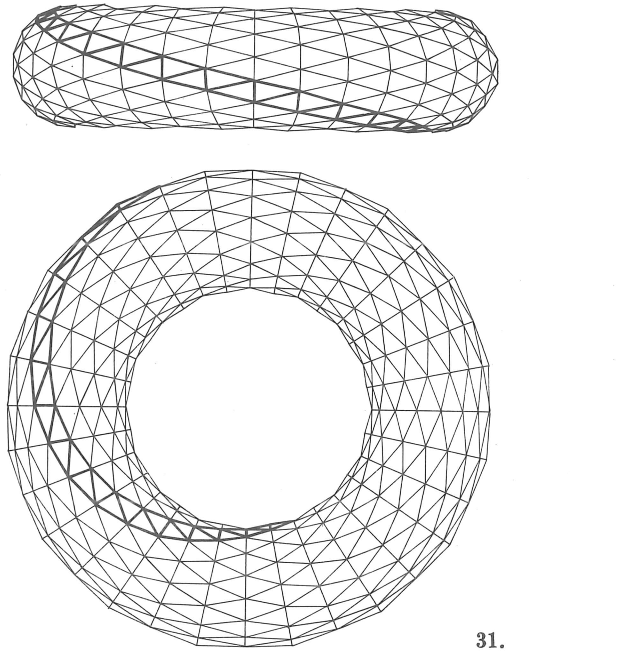


30.

PLATE VII. TORUS—DIVISION OF SURFACE WITH CHORDS AND/OR SEGMENTS. *(CHORD: The chord of a circular arch. SEGMENT: The chord of a a non-circular arch.)* 27. TRIANGLES. A variation of Fig. 18 and Fig. 23. Equal length segments between circular sections. Chords between circular sections of different length. 28. DIAMONDS. Segments determined by the intersection of two equal closed helices of opposite hand. 29. DIAMONDS. Segments determined by the intersection of Lemniscates. (See PLATE III CC) 30. DIAMONDS. Similar to Fig. 21. Segments determined by the intersection of Cassinian curves. (See Plate III BB).



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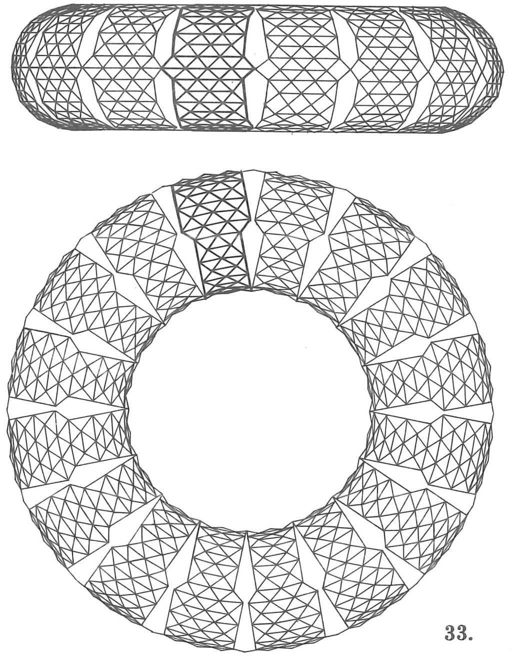

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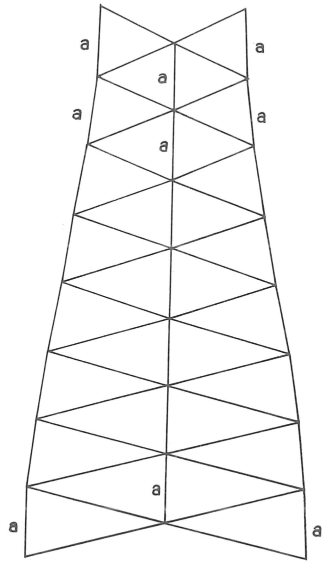

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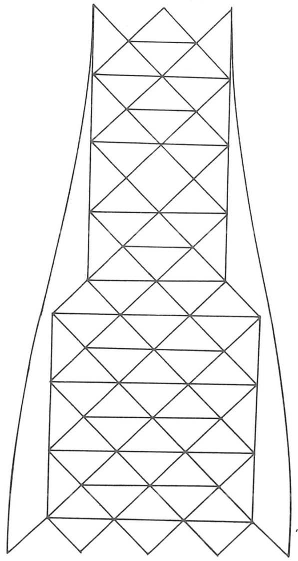
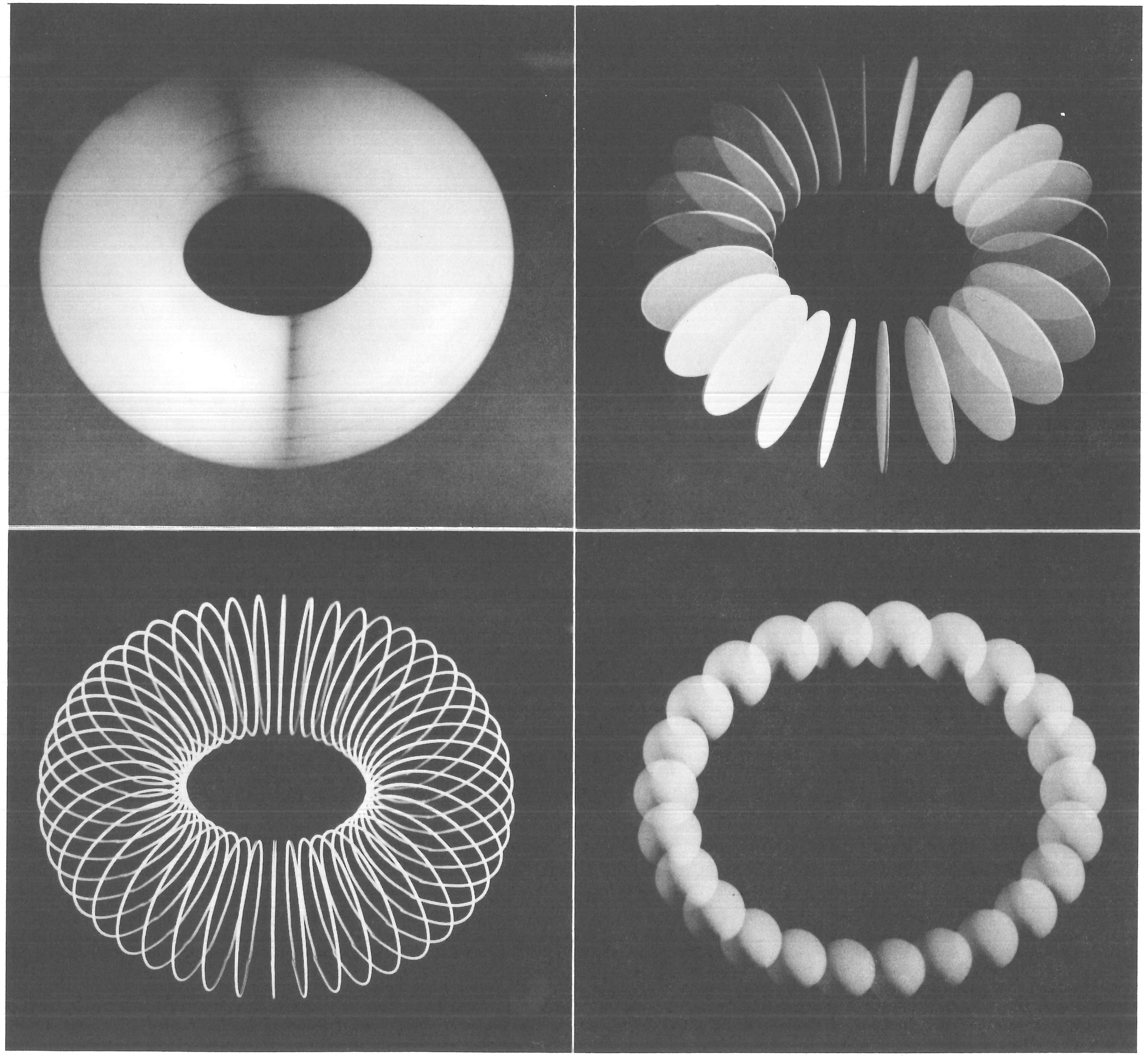


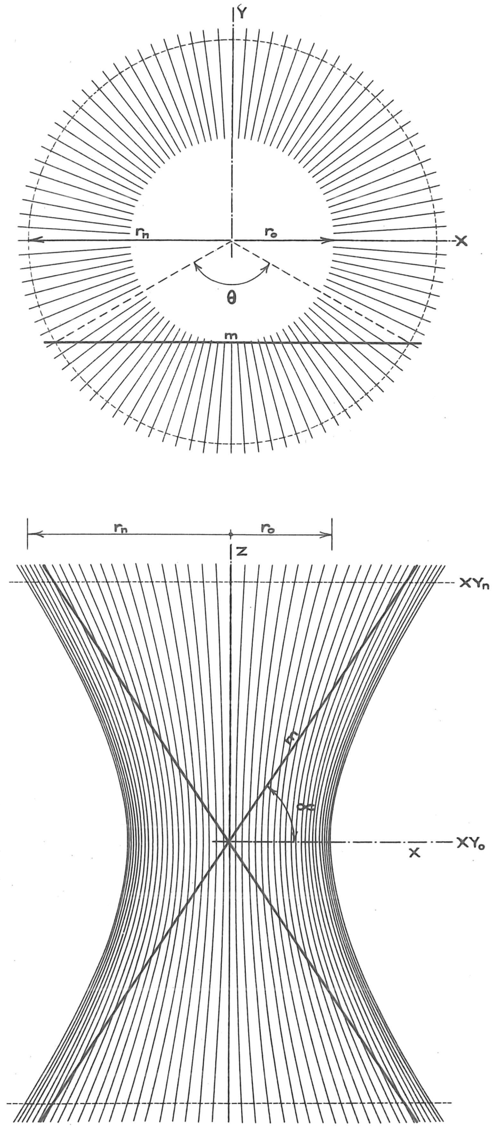
34.

PLATE VIII. TORUS—DIVISION OF SURFACE WITH CHORDS AND/OR SEGMENTS. *(CHORD: The chord of a circular arch. SEGMENT: The chord of a circular arch)* 31. TRIANGLES. Same as Fig. 24. 32. Unrolled plan of a radial sector. 33. TRIANGLES. The torus is composed of a series of cylinders. Each cylinder is divided into isosceles triangles. 34. Unrolled plan of a cylinder.



35. 37. 36. 38.

PLATE IX. TORUS Dupin Cyclide. 35. Time exposure of a rotating circle. 36-37. Multiple exposure of a rotating circle. 38. Multiple exposure of a rotating sphere.



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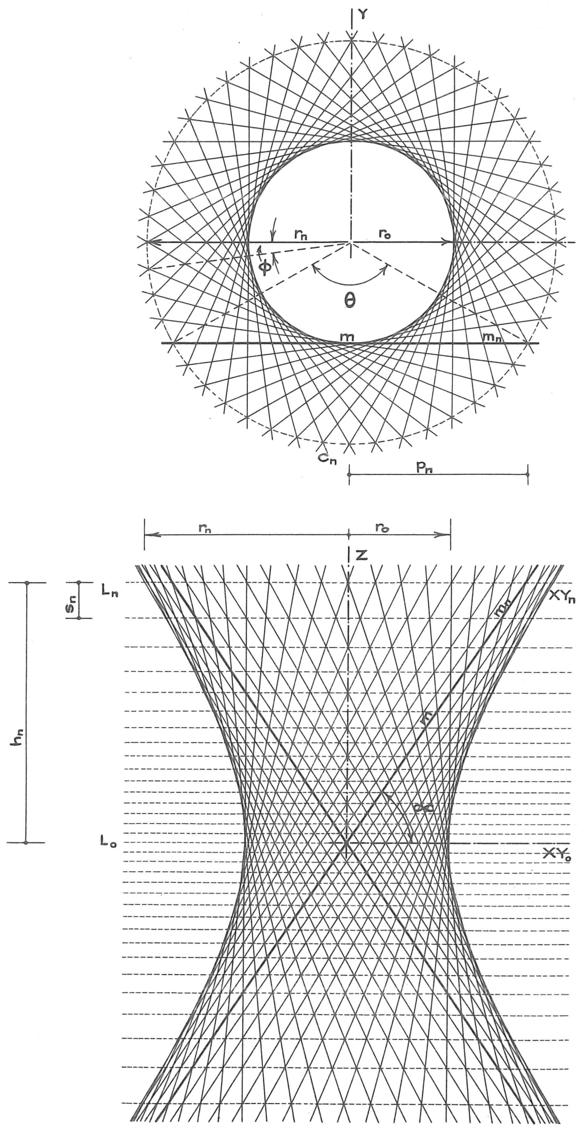
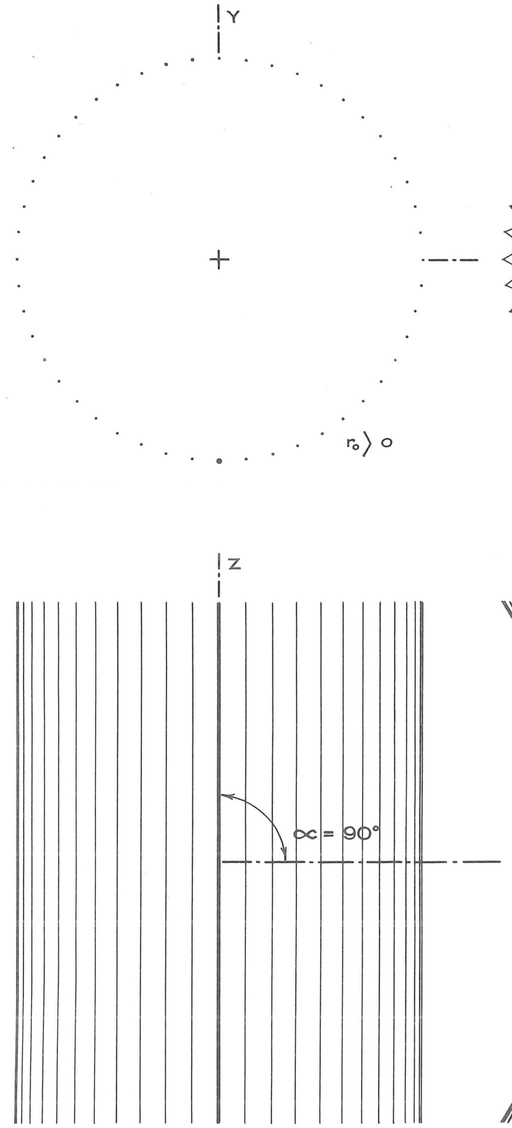


40.

PLATE X. HYPERBOLOID OF REVOLUTION OF ONE SHEET—GENERATION—NOTATION 39. The locus described by a hyperbola revolving around its conjugate axis. 40. The locus described by a straight generatrix revolving around a non-parallel, non coplanar axis.



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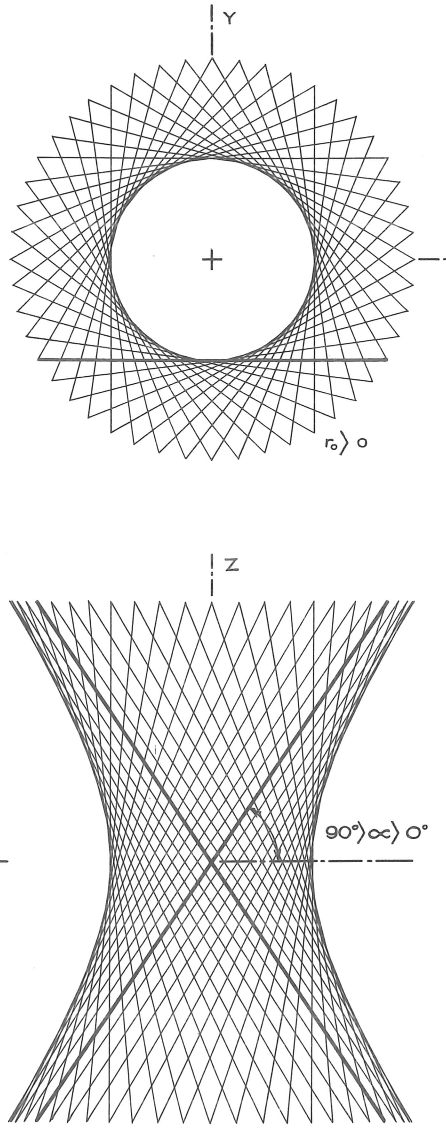

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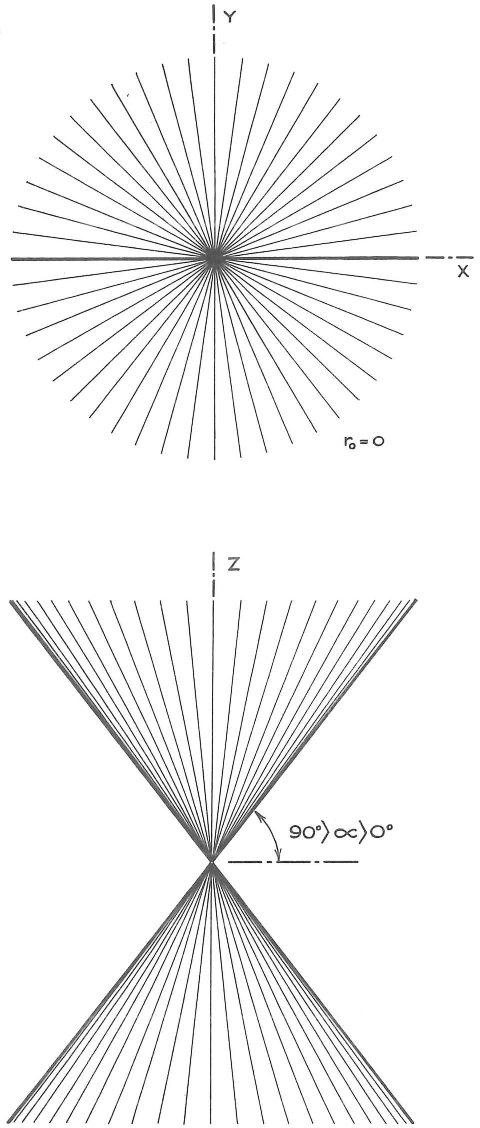

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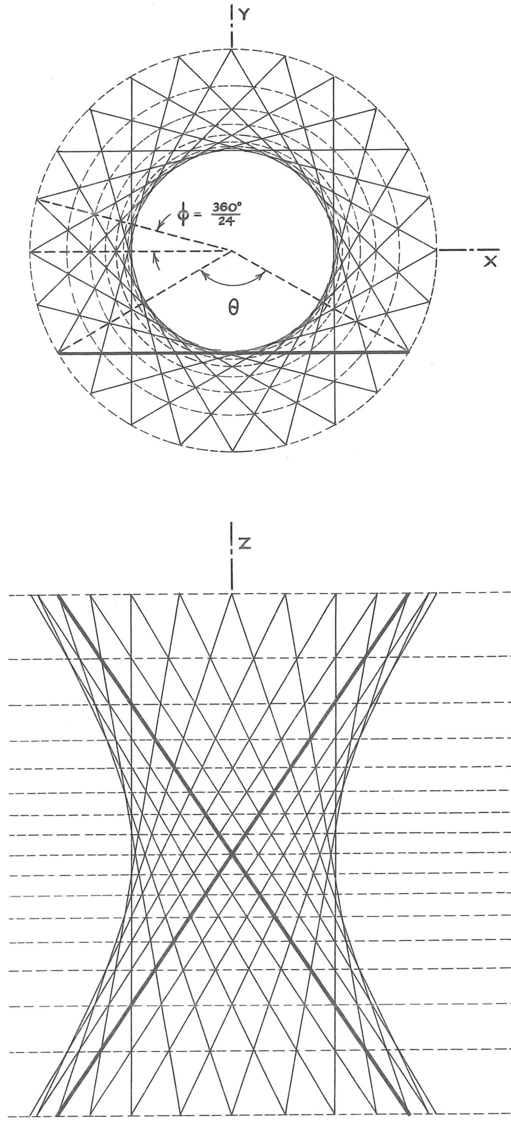

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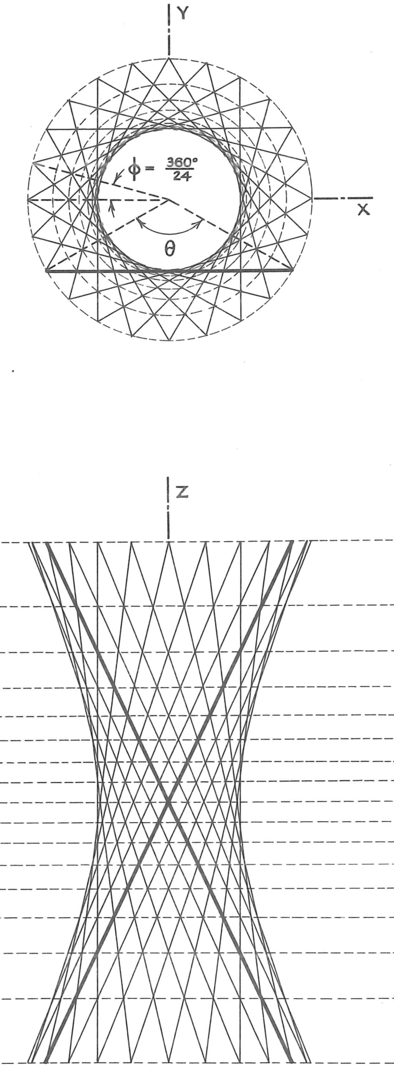

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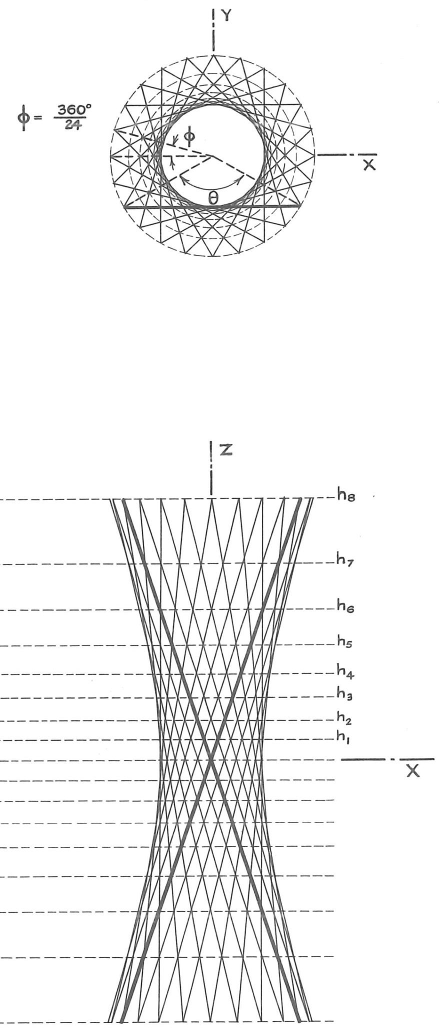
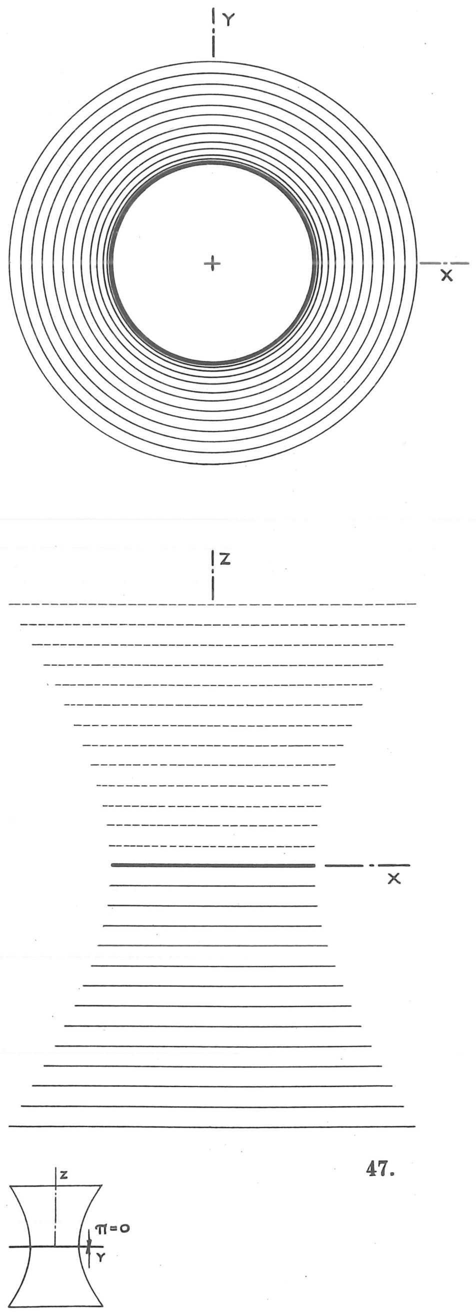


46.

PLATE XI. HYPERBOLOID—DEFINING CONDITIONS 41. $\alpha = 90°$; $r_o > 0$ CYLINDER. 42. $90° > \alpha > 0°$; $r_o > 0$ HYPERBOLOID OF REVOLUTION. 43. $90° > \alpha > 0°$; $r_o = 0$ INVERTED CONES.

PLATE XII. HYPERBOLOID—"PARALLEL" 44, 45 46. ILLUSTRATE THE FOLLOWING PROPERTY: All the hyperboloids, with the same angle Θ, of rotation at the same n level, have all the generatrices frozen at equal intervals $\frac{360°}{N} = \phi$, intercepting at the same levels : $h_o, h_1, h_2 \dots h_n$.



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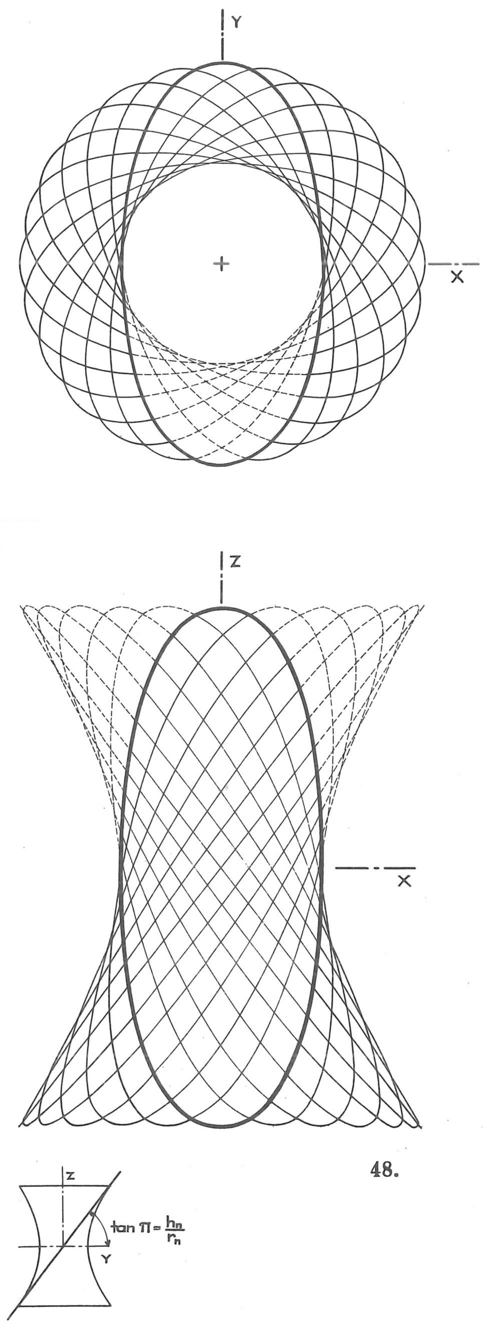

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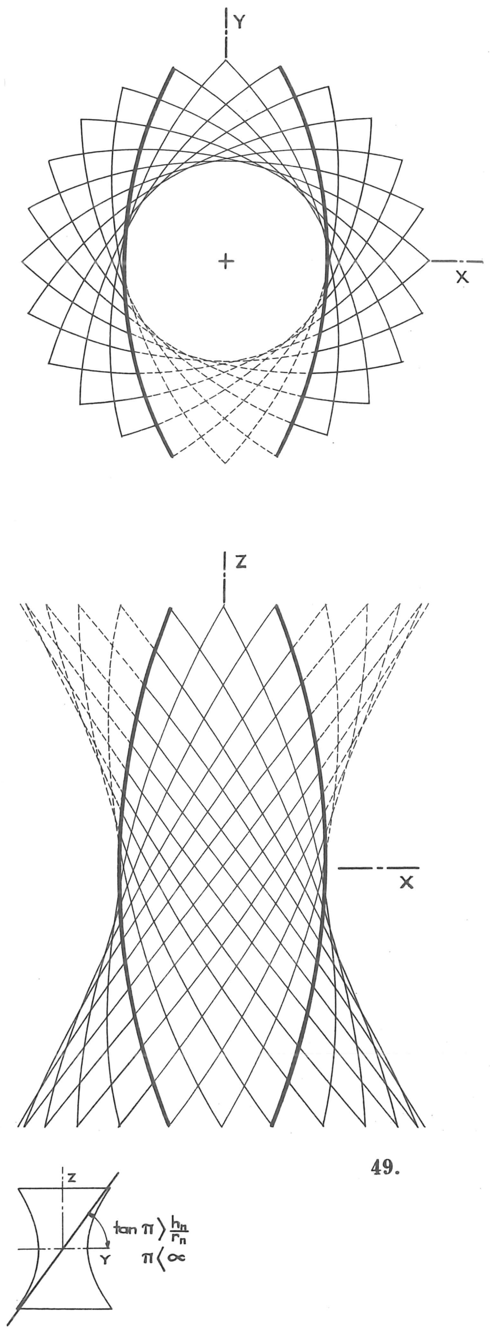
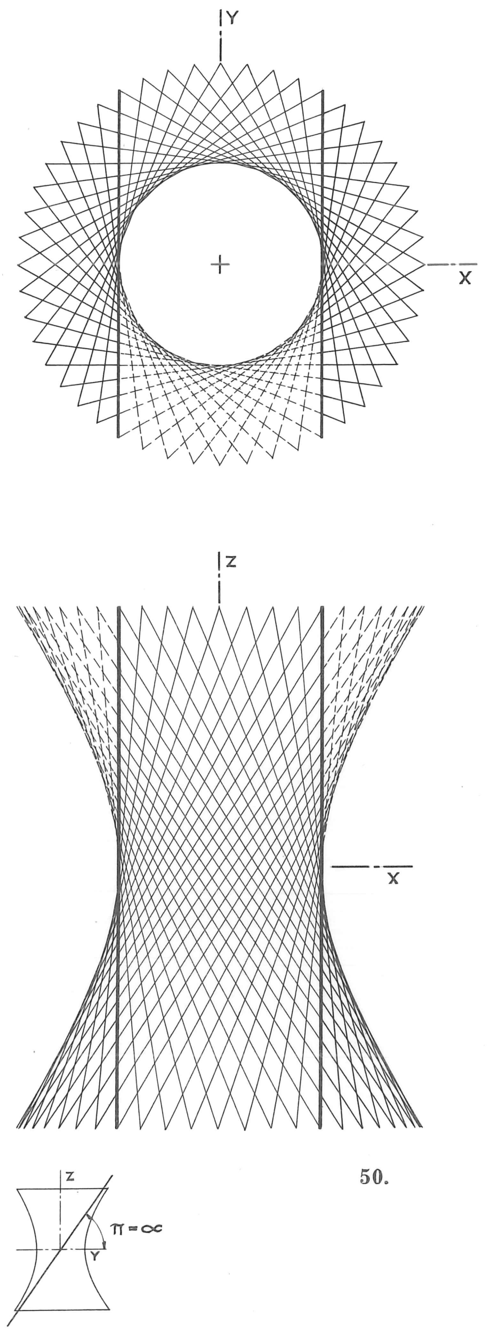


49.

PLATE XIII. HYPERBOLOID—TRANSLATION AND ROTATION OF PLANE SECTIONS 47. TRANSLATION. Planes perpendicular to Z axis. Sections: circles. 48. ROTATION. Trace of plane through X axis. Angle of inclination π. $\tan \pi = \frac{h_n}{r_n}$ Section. A cassinian curve. 49. ROTATION. Trace of plane through X axis. Angle of inclination π. Tan $\pi > \frac{h_n}{r_n}$; $\pi = a$ Section: A segment of cassinian curve.



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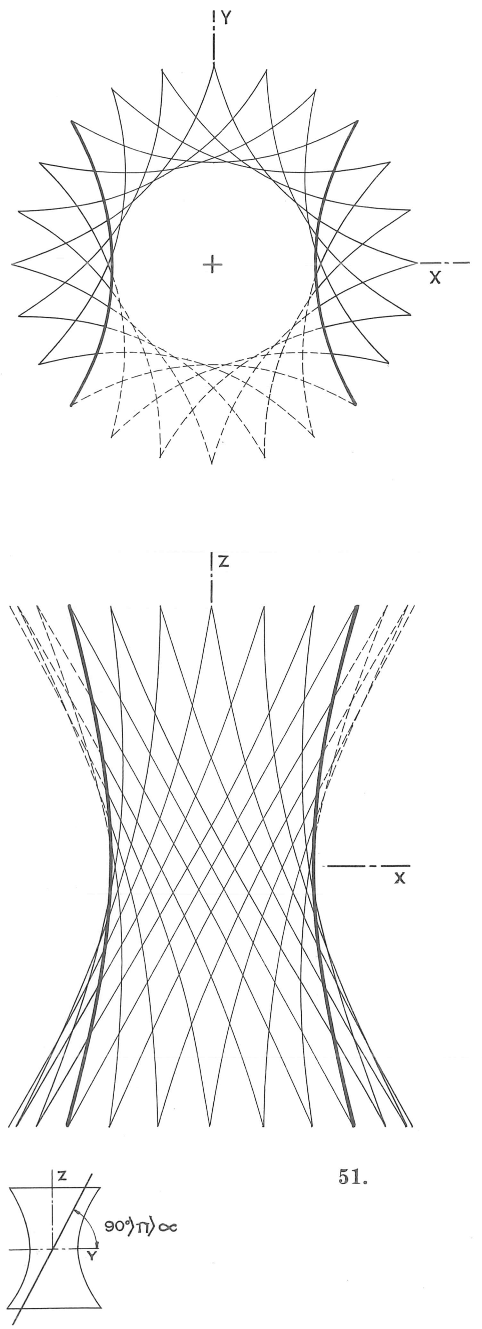

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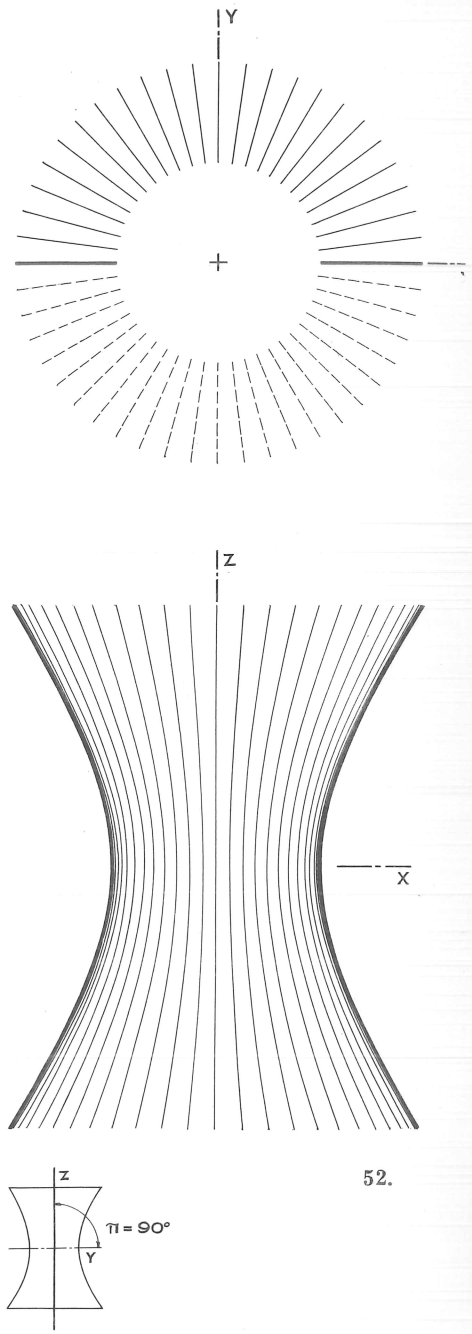
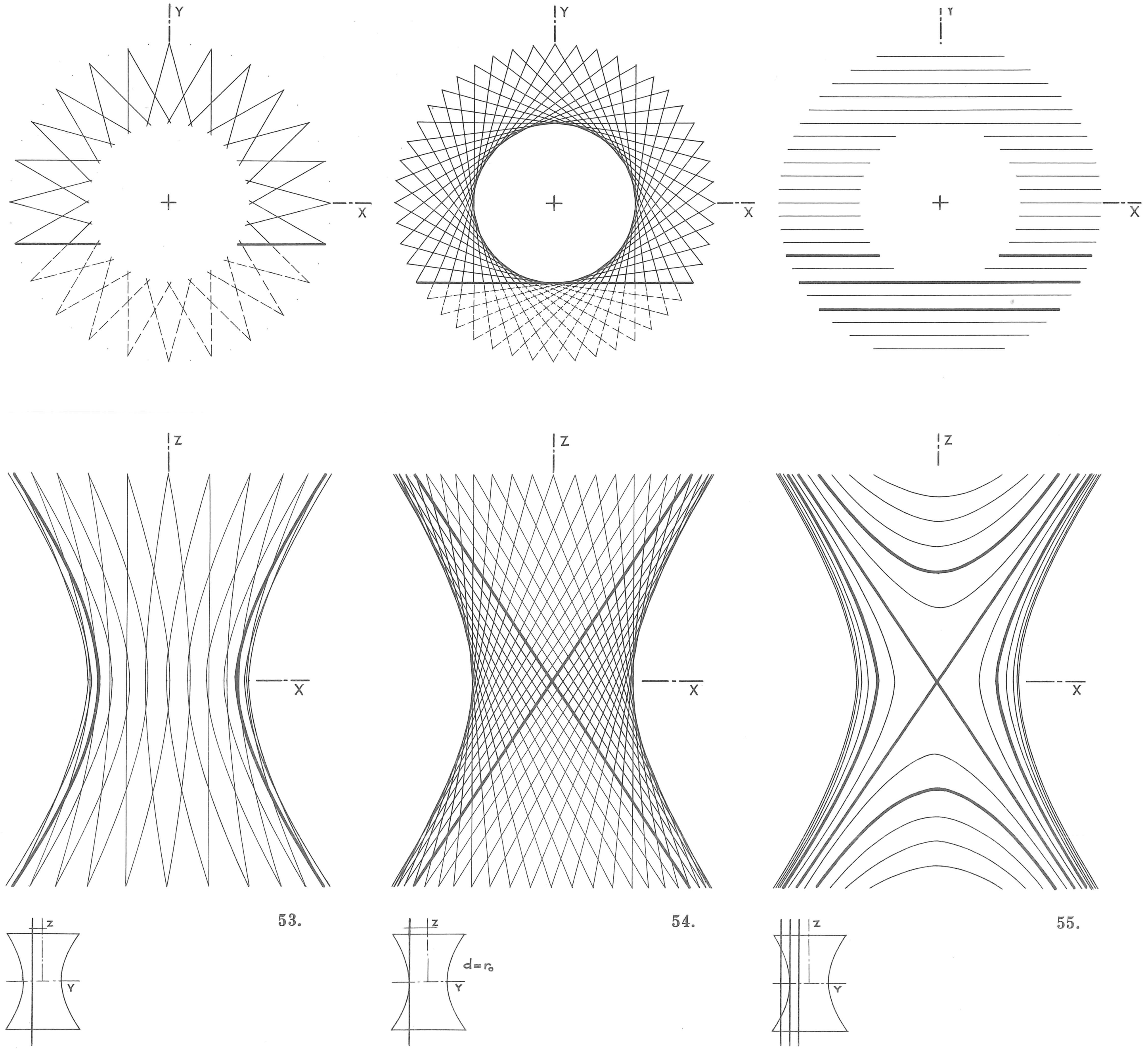


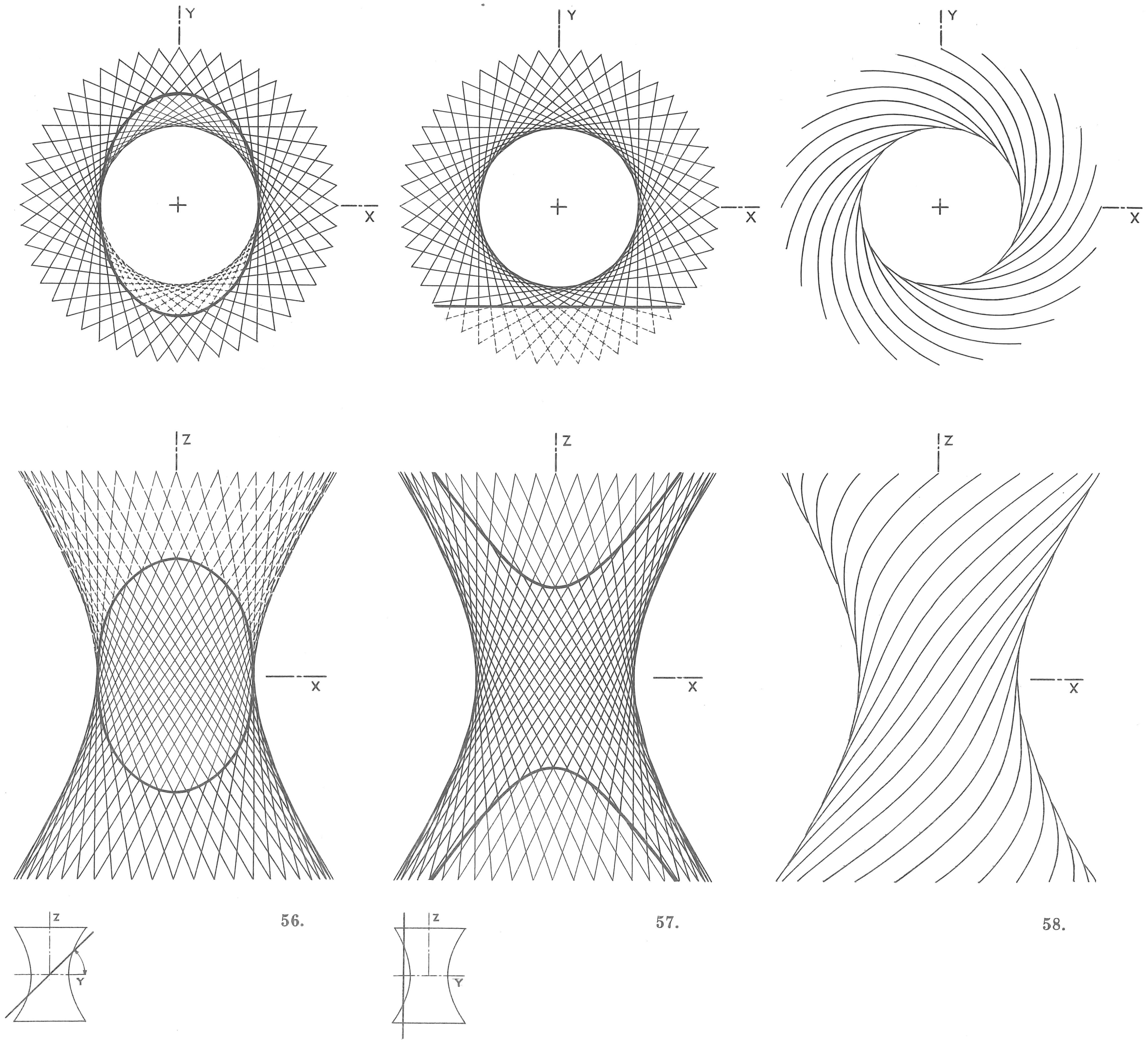
52.

PLATE XIV. HYPERBOLOID—ROTATION OF PLANE SECTIONS 50. Trace of plane through X axis. Angle of inclination $\pi = a$ Section: Two parallel lines. 51. Trace of plane through X axis. Angle of inclination π. $90° > \pi > a$ Section: Two hyperbolas. 52. Trace of plane through X axis. Angle of inclination $\pi = 90°$ Section: Two hyperbolas.



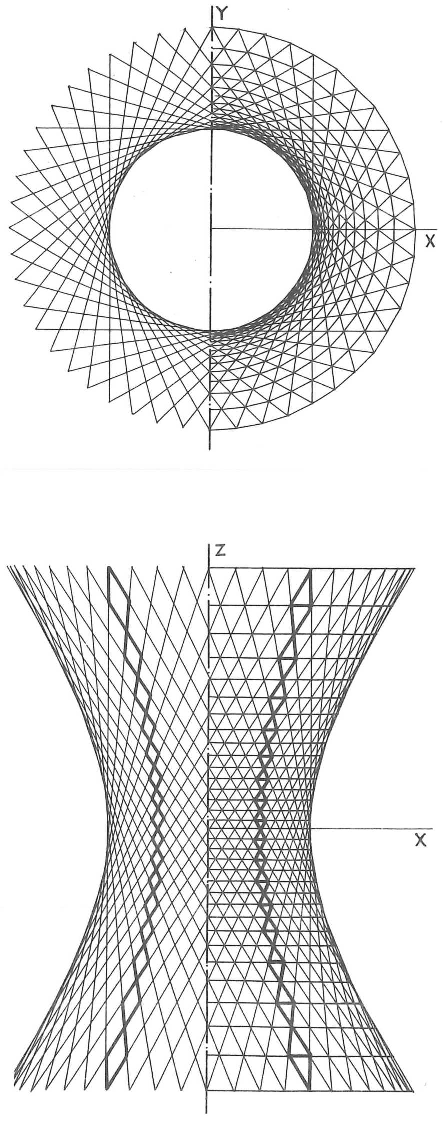
53. 54. 55.

PLATE XV. HYPERBOLOID—TRANSLATION AND ROTATION OF PLANE SECTIONS 53. ROTATION. Plane parallel to Z axis — Distance $d : r_o > a > 0$ Section: Two hyperbolas. 54. ROTATION. Plane parallel to Z axis. Distance $d = r_o$ Section: Two straight generatrices. 55. TRANSLATION. Planes parallel to Z axis. Sections are a series of conjugate hyperbolas (see Figs. 52, 53, 57) with the same asymptotes (see Fig. 54)



56. 57. 58.

PLATE XVI. HYPERBOLOID—ROTATION OF SECTIONS 56. Trace of plane through X axis. Angle of inclination π, from $\pi = 0°$ to $\text{Tan } \pi < \frac{h_n}{r_n}$ Section: A cassinian curve. 57. Compare with Fig. 55. Plane parallel to Z axis. Distance $d < r_o$ Section: Two hyperbolas. 58. Helicoidal section.



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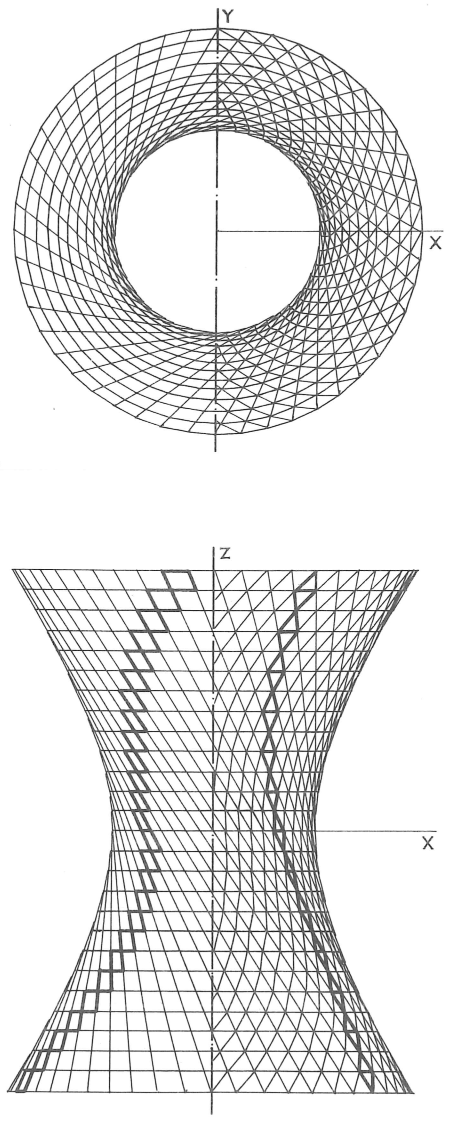

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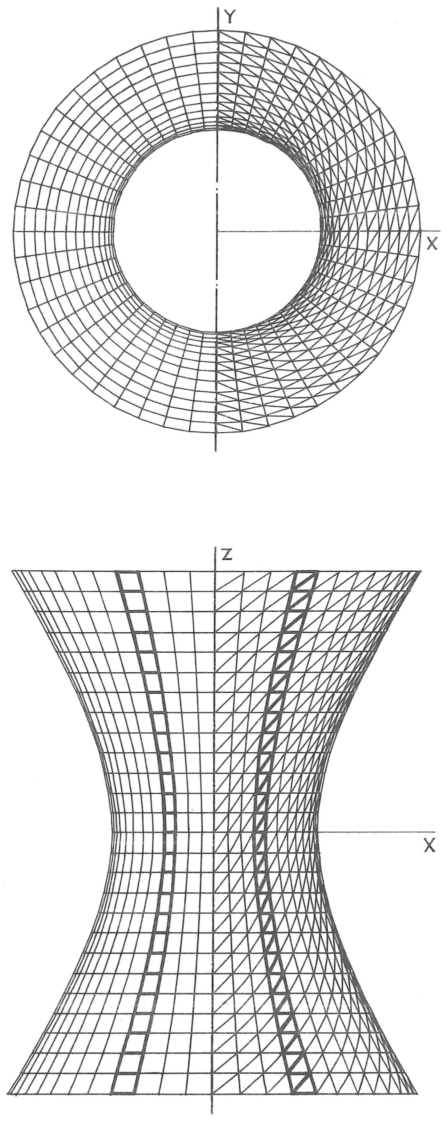
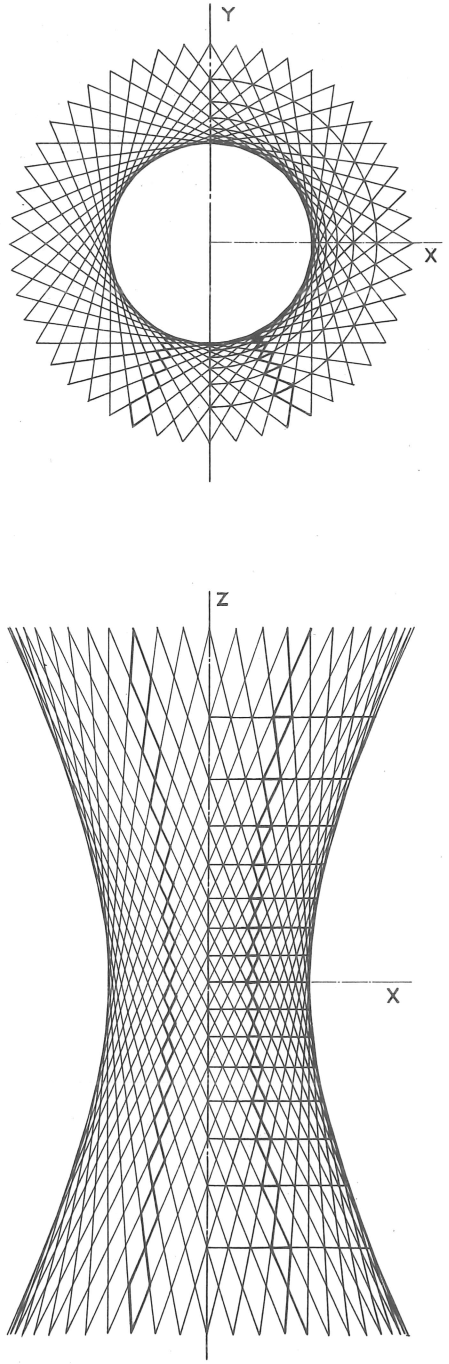
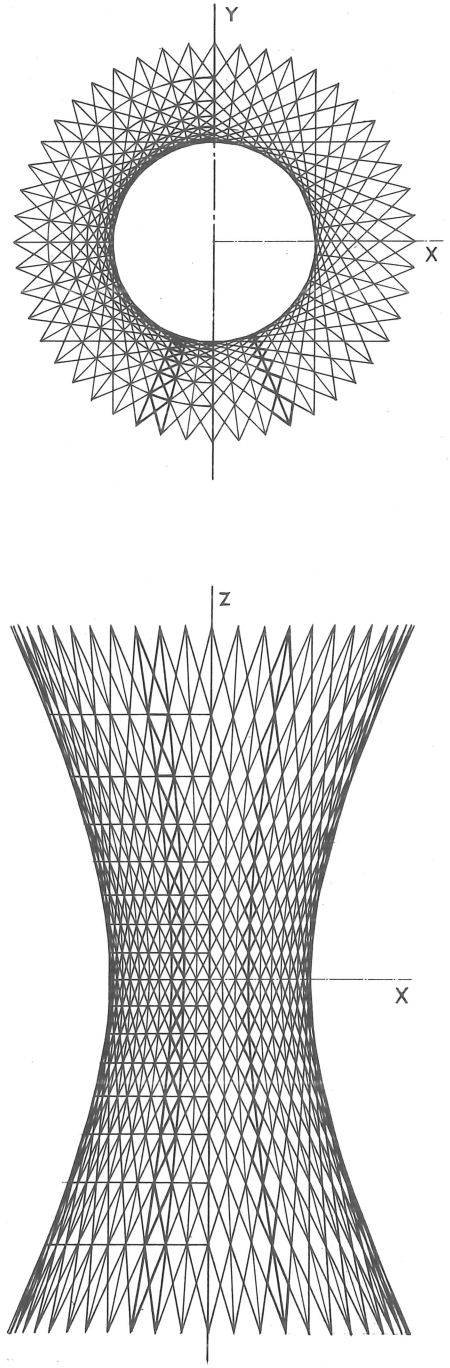


61.

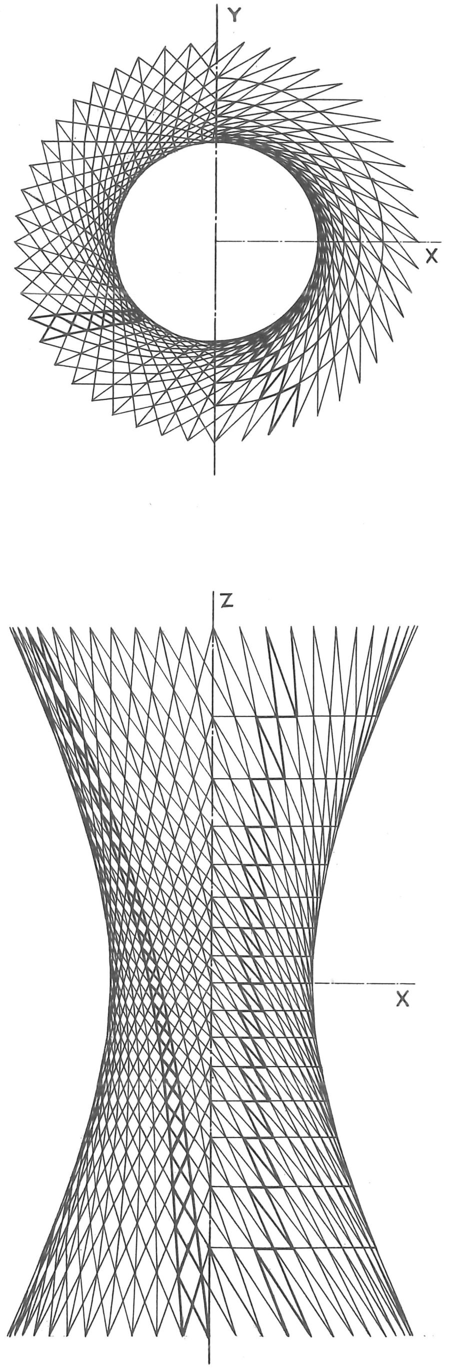
PLATE XVII. HYPERBOLOID—DIVISION OF SURFACE 59. DIAMONDS AND TRIANGLES. Division determined by the intersection of pairs of straight generatrices, frozen at equal intervals: $\frac{360^o}{N} = \phi$ 60. DIAMONDS AND TRIANGLES. Division determined by the intersection of straight generatrices frozen at equal intervals: $\frac{360^o}{N} = \phi$ and circular sections at equal intervals. 61. DIAMONDS AND TRIANGLES. Division determined by the intersection of hyperbolic generatrices frozen at equal intervals and circular sections at equal intervals.



62.

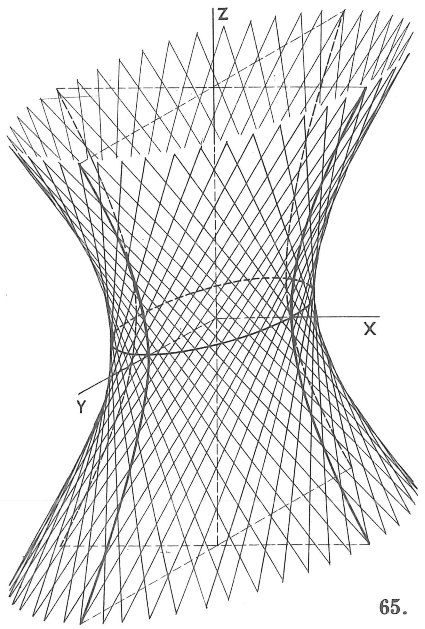


63.



64.

PLATE XVIII. HYPERBOLOID—DIVISION OF SURFACE 62, 63, 64. DIAMONDS AND TRIANGLES. Variations of divisions illustrated on Fig. 59.



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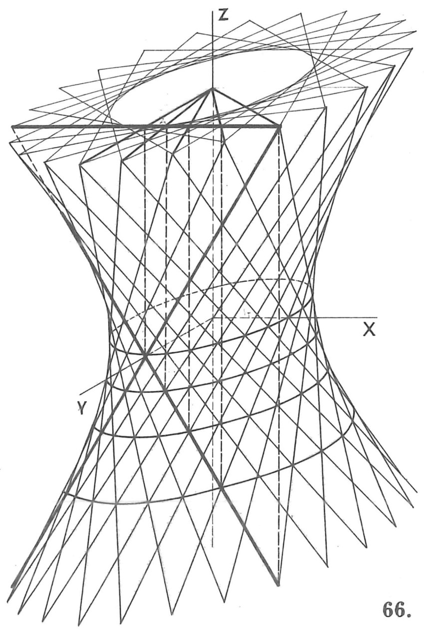

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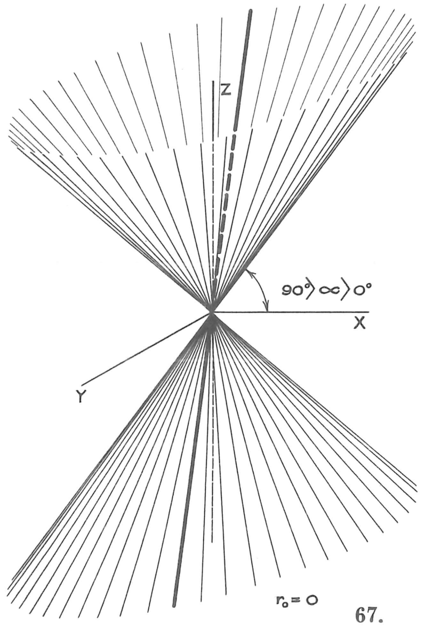

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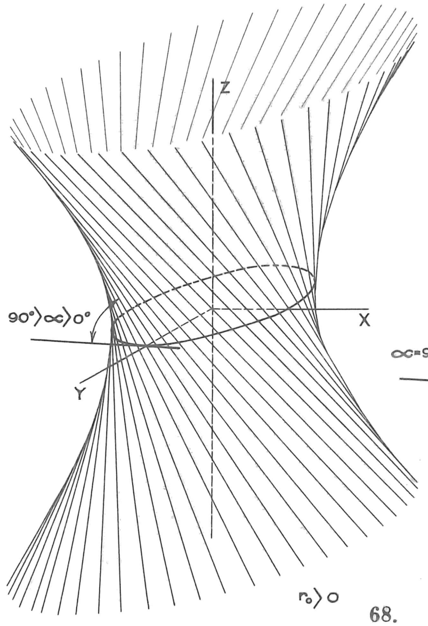

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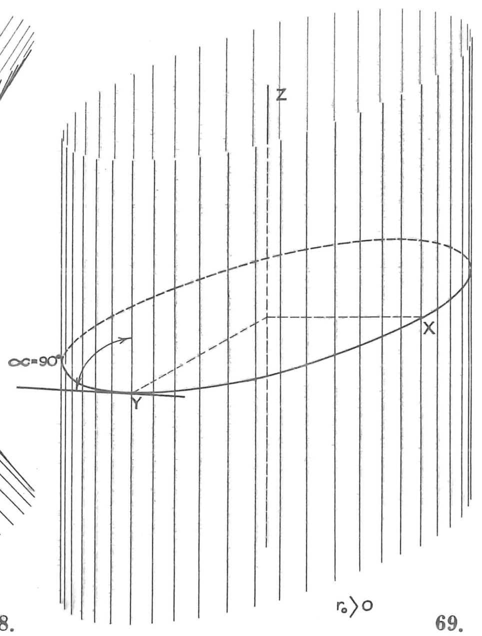
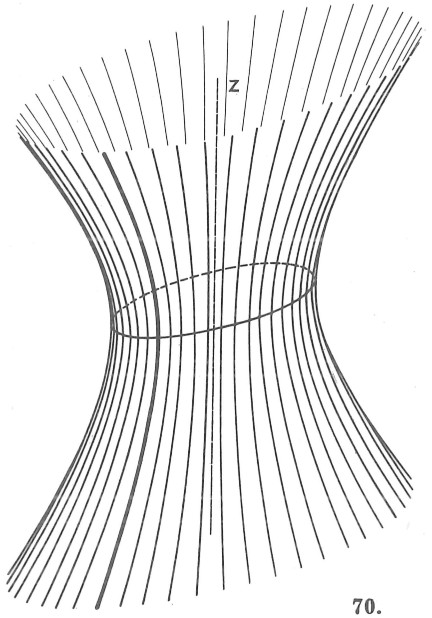


69.

PLATE XIX. HYPERBOLOID—ISOMETRIC VIEWS 65, 66. Illustrates characteristics planes, angles and lines. Compare with Fig. 40. 67, 68, 69. Same as Figs. 41, 42, 43



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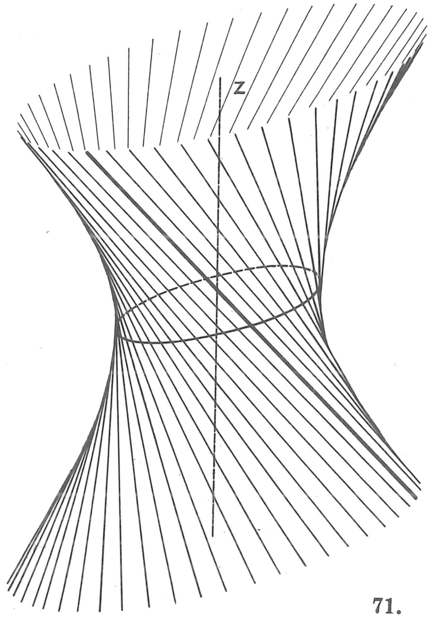

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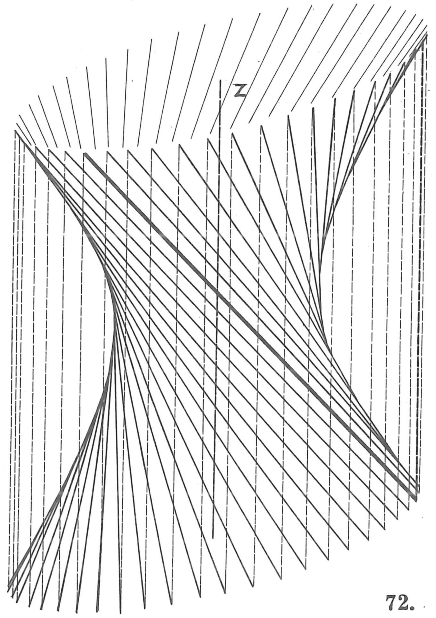
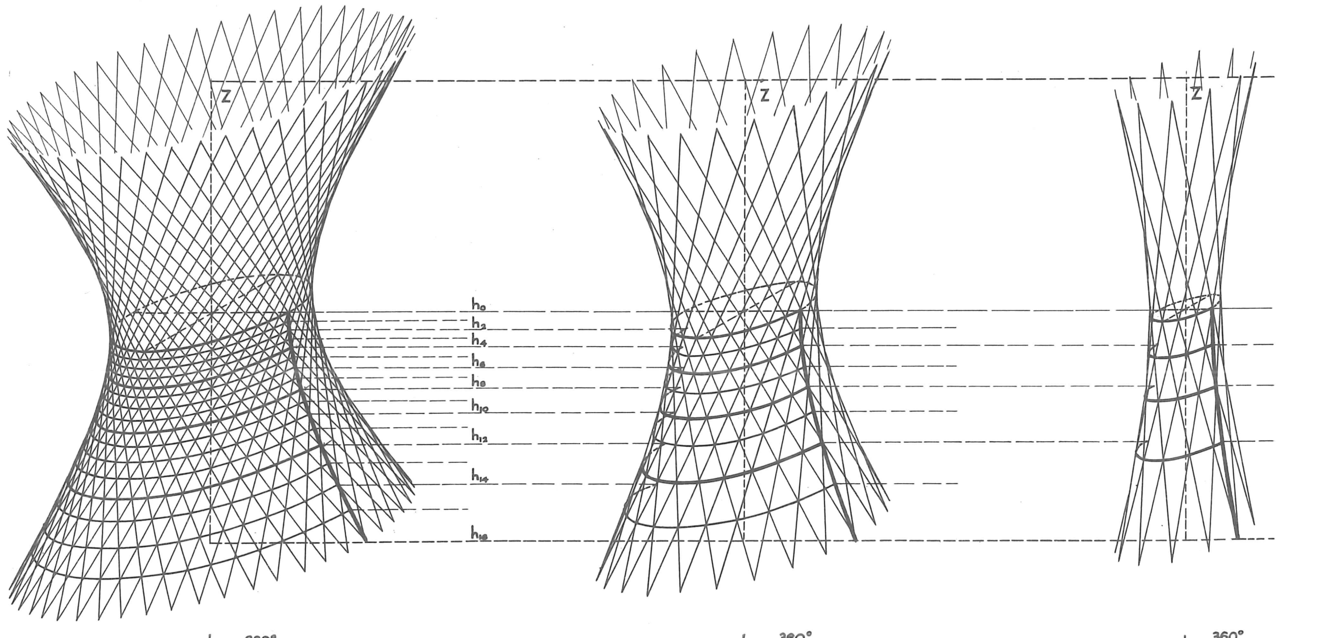


72.

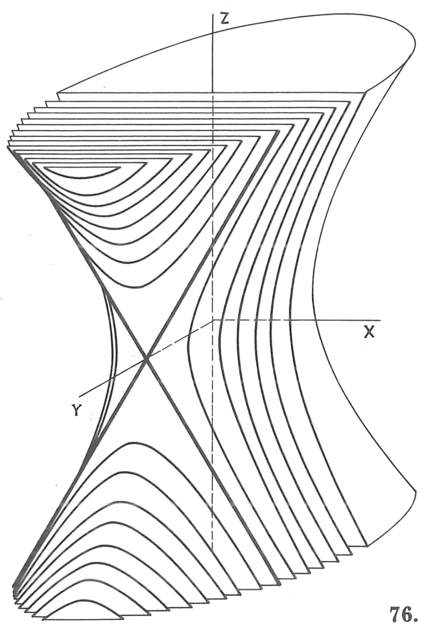


$\phi = \frac{360°}{48}$ 73.

$\phi = \frac{360°}{24}$ 74.

$\phi = \frac{360°}{12}$ 75.

PLATE XX. HYPERBOLOID—ISOMETRIC VIEWS 70. Generation by hyperbolic generatrices 71. Generation by straight line generatrices 72. Generation by rotation of straight line generatrices of a cylinder. 73, 74, 75. Same as Figs. 44, 45, 46



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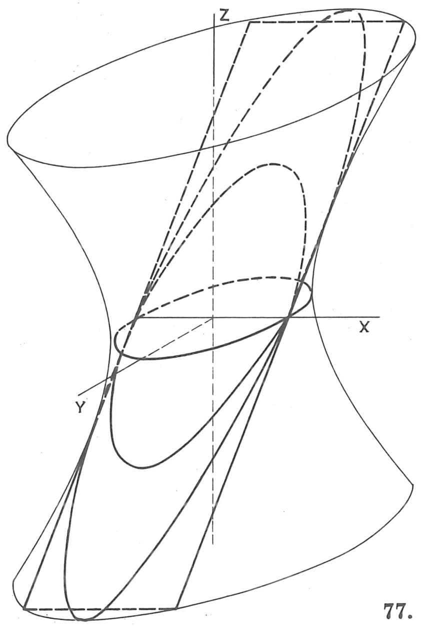

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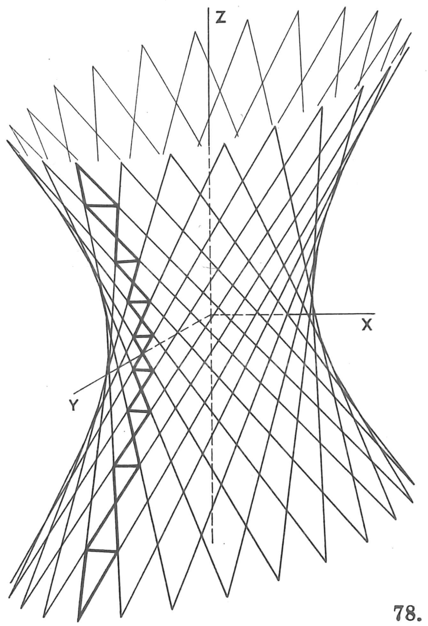

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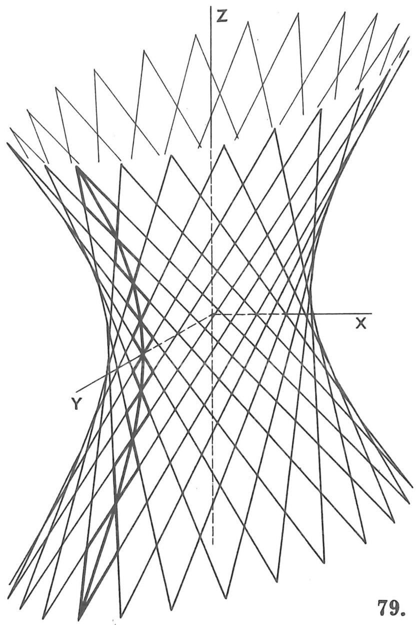

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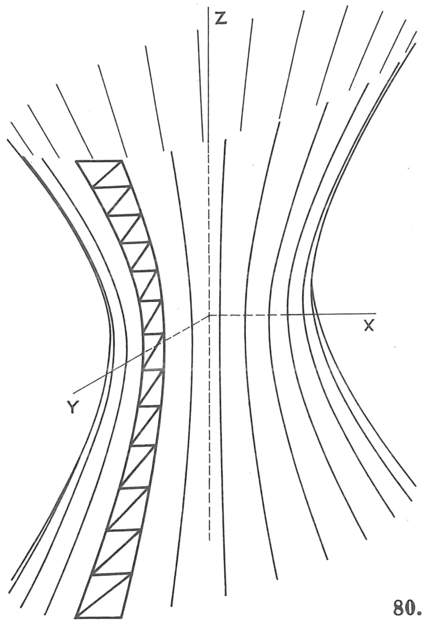
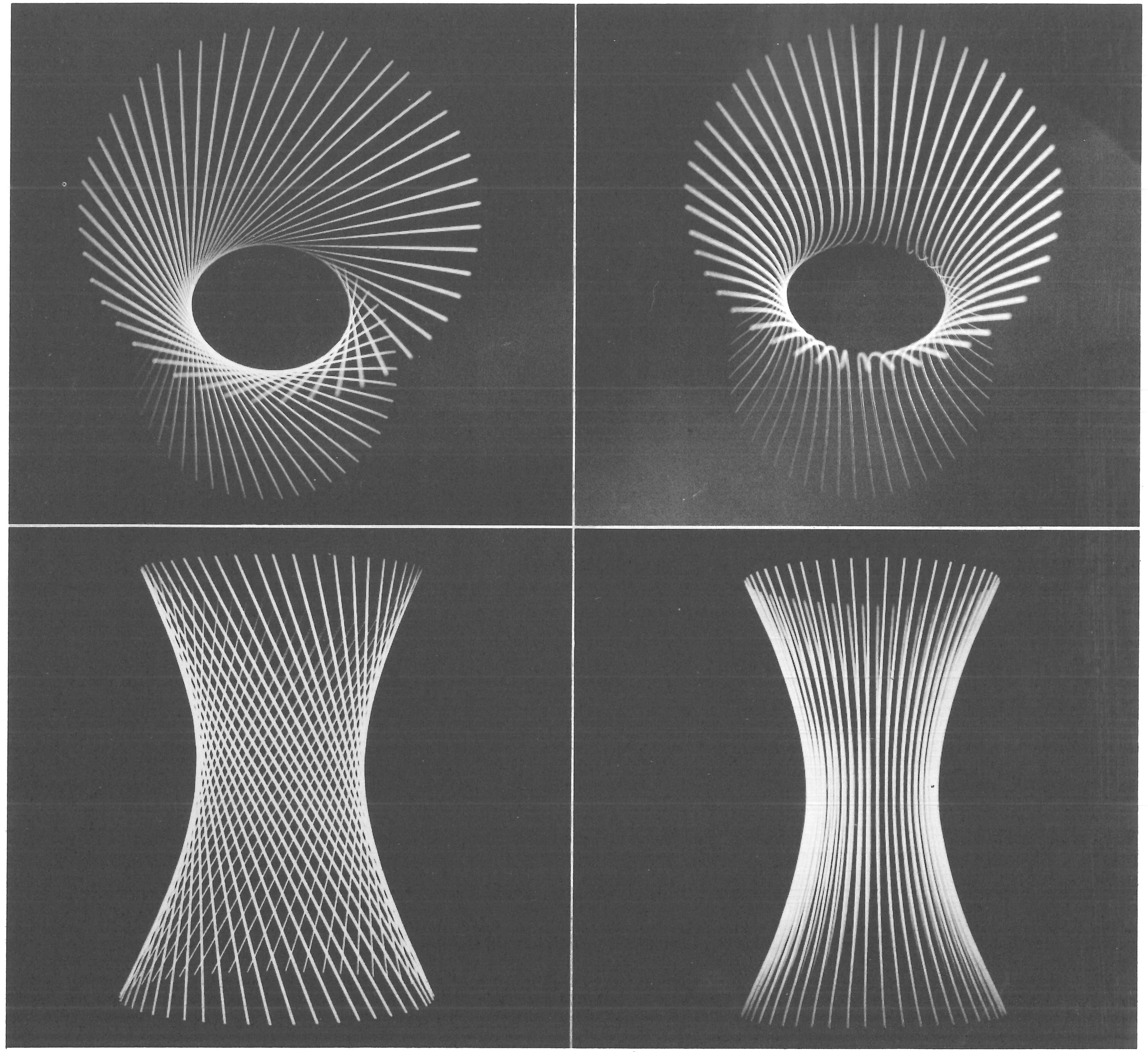


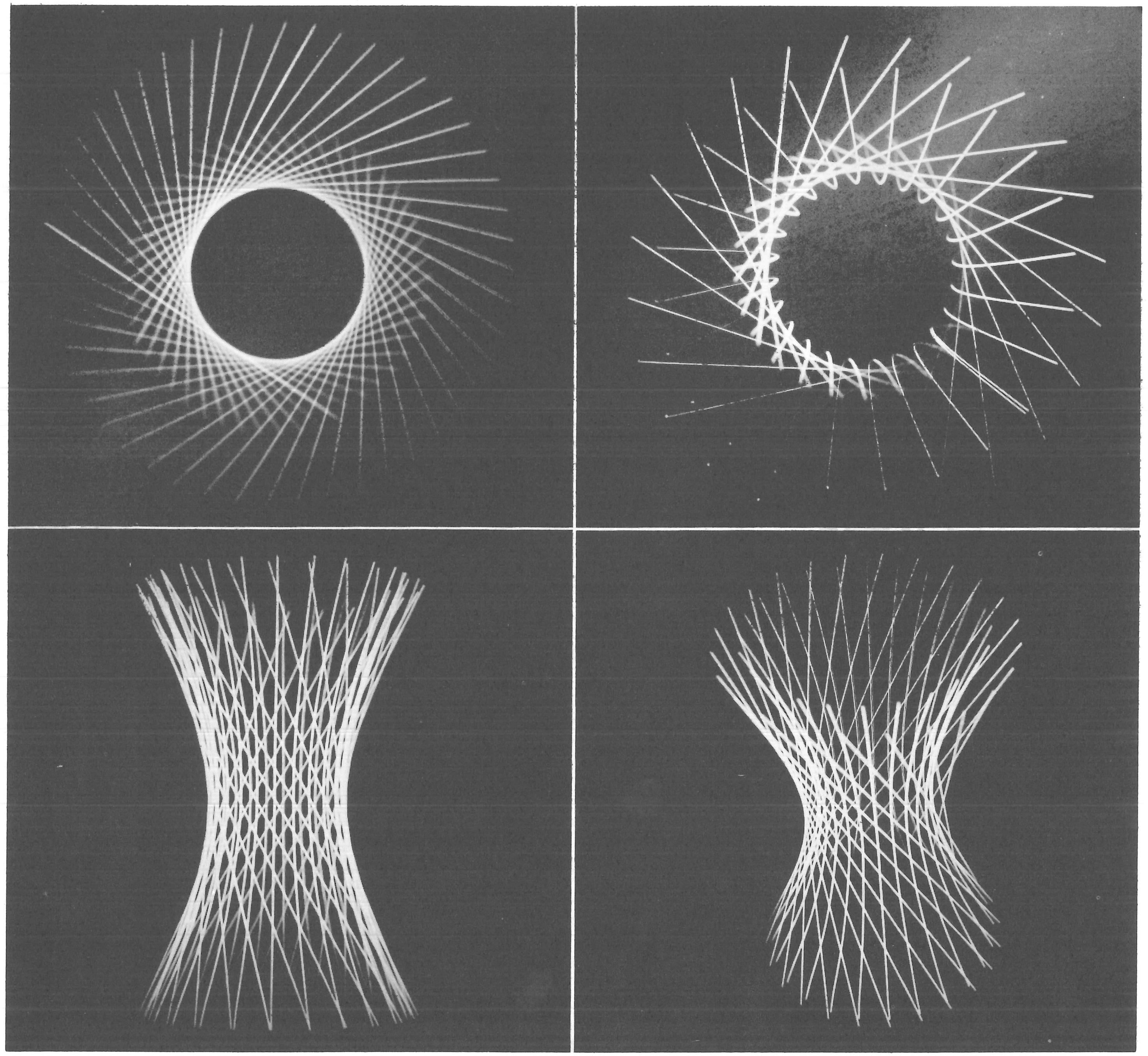
80.

PLATE XXI. HYPERBOLOID—ISOMETRIC VIEWS 76. Same as Fig. 55 77. Sections of Figs. 47, 48, 50, 56 78. Same as Fig. 59 79. Same as Fig. 63 80. Same as Fig. 61



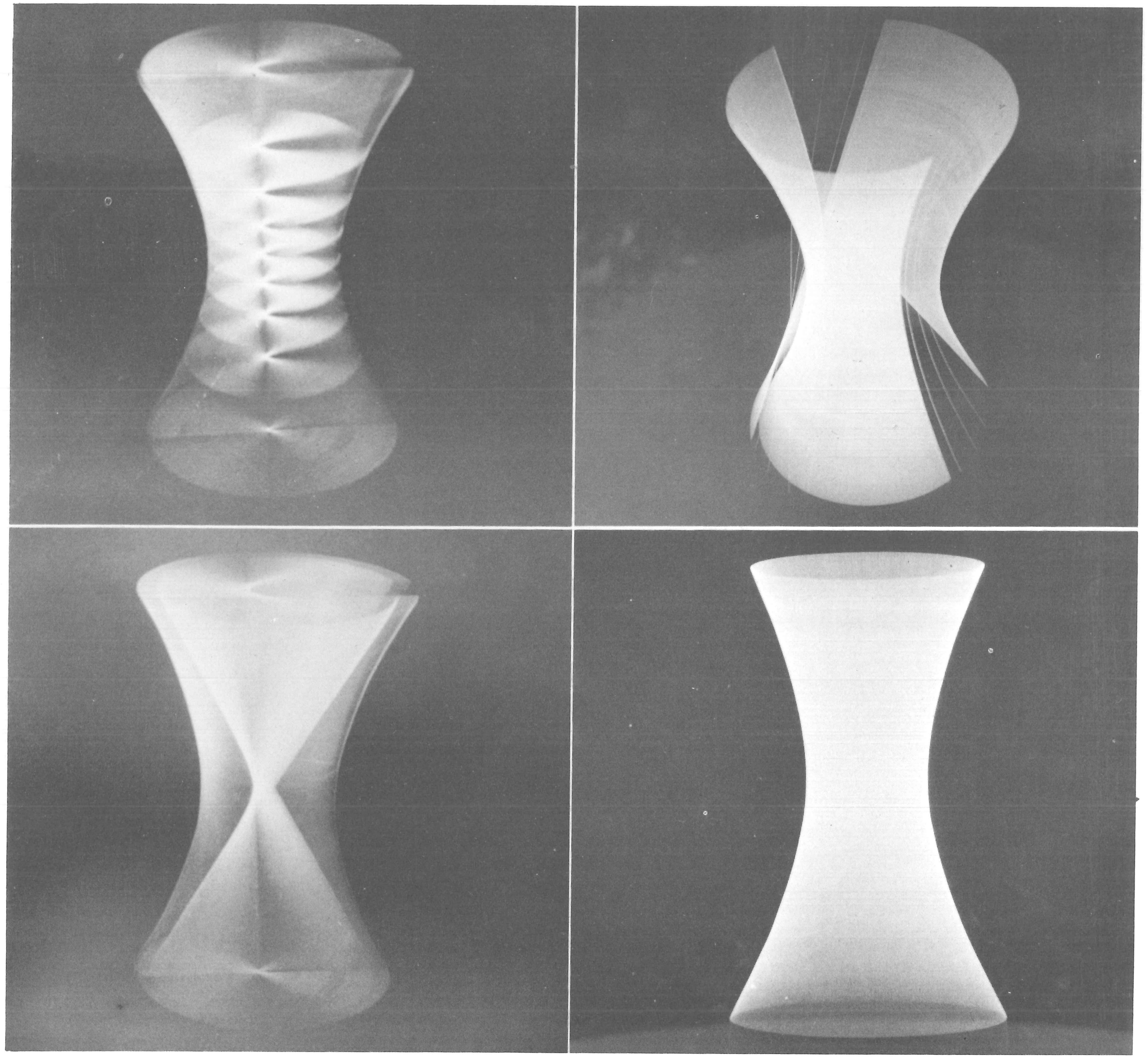
81. 83. 82. 84.

PLATE XXII. HYPERBOLOID 81, 82. Multiple exposure of rotating straight generatrix. 83, 84. Multiple exposure of rotating hyperbola.



85. 87. 86. 88.

PLATE XXIII. HYPERBOLOID 85. Multiple exposure of rotating straight generatrix. 86, 87, 88. Multiple exposure of rotating straight generatrix and hyperbola.



89. 91. 90. 92.

PLATE XXIV. HYPERBOLOID 89. Time exposure of rotating radii. 90. Time exposure of rotating straight gneratrix and hyperbola. 91. Time exposure of rotating hyperbola and asymptotes. 92. Time exposure of rotating hyperbola.