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STRUCTURES OF WARPED SURFACES

COMBINATIONS OF UNITS OF HYPERBOLIC PARABOLOIDS

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INTRODUCTION

The Hyperbolic Paraboloid, a double-curved surface generated by the displacement of a straight line, and commonly described as a saddle shape, has become the unit-theme of many structures built all over the world during the last decade. The first known structural development based upon such units was introduced in France by Bernard Laffaille, who in 1933, built at Dreux the two-sided cantilevered structure shown in Figure 1. As the result of such experimental work, he published two years later the *"Memoirs sur L'Etude Generale des Surfaces Gauches"* in the journals of the Second Congress of the International Association of Bridges and Structural Engineering.

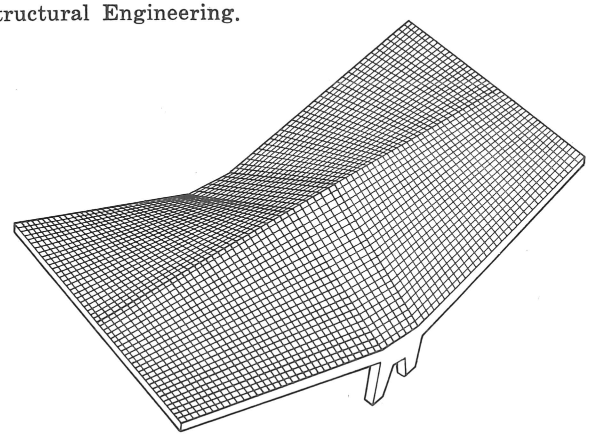


Figure 1

In 1936, the French engineer F. Aimond published the most complete study ever made on the subject, in the fourth volume of the above mentioned journal, called *"Treatise on the Statics of Hyperbolic Paraboloid Shells not Stiff in Bending."* This study covers a structural analysis of these warped surfaces, as well as both simple and elaborate geometrical combinations of Hyperbolic Paraboloid units to enclose varied spaces. During the same year, L. Issenmann Pilarski, in his book *Calcule des Voiles Minces en Beton Arme,* published by Dunod in Paris, France, included part of the studies made by Laffaille and Aimond, thus completing the original bibliography on the subject.

Although the Hyperbolic Paraboloid had been well known as a geometric surface, it was not used until 1933 as a structure. Only Antonio Gaudi, the Spanish architect, saw the architectural and structural possibilities of such surfaces, before Laffaille and Aimond. In the basement of La Sagrada Familia, Gaudi's unfinished church in Barcelona, Spain, there are two plaster models of structures formed by three rhomboidal units of Hyperbolic Paraboloids, combined in a hexagonal plan. They are advanced for the period in which they were conceived, and constitute perhaps the best examples of Gaudi's structural ideas.

After the first structures were built by Laffaille, the Italian engineer Giorgio Baroni built several industrial roofs using units of Hyperbolic Paraboloids for the Alfa Romeo factory in Italy. The impact of these constructions was reduced, unfortunately, due to the outbreak of the Second World War, which completely paralyzed all civil construction in Europe.

The rebirth of the Hyperbolic Paraboloid came in 1950, when it was used as a saddle-shape in the Cosmic Ray Pavilion of the University City of Mexico. This construction constitutes the beginning of uninterrupted structural developments by the Spanish architect, Felix Candela. With the persistence present in the best builders' tradition, and with his constant exploration of the structural and visual richness of the Hyperbolic Paraboloid, he has made a lasting contribution to the art of construction.

The material published here is based upon studies made by the author since 1952 as a part of his courses in architectural design at the School of Design in Raleigh, North Carolina, and later at the School of Architecture and Planning at the Massachusetts Institute of Technology in Cambridge. Some of these studies were developed by students in their attempt to understand the geometric, structural, and architectural characteristics that result from the combination of Hyperbolic Paraboloid units, through the construction of models in varied materials, techniques, and scales.

The accompanying plates attempt to convey to the reader how combinations of these nonplanar, four-sided surfaces of great structural efficiency can create almost endless architectural spatial relationships.

We hope that those who may be interested in these forms for architectural use do not blindly translate them into buildings. Here, they purposely have been represented *solely* as three dimensional organizations of the four-sided units, independent of the lengths of their sides; of the angles formed by their sides; of their curvatures; materials and surface treatment; and independent of their scale in relation to any element of reference or to any given environment. Misinterpretation of these variables undoubtedly will transform a potentially valid visual event into an actual visual offense.

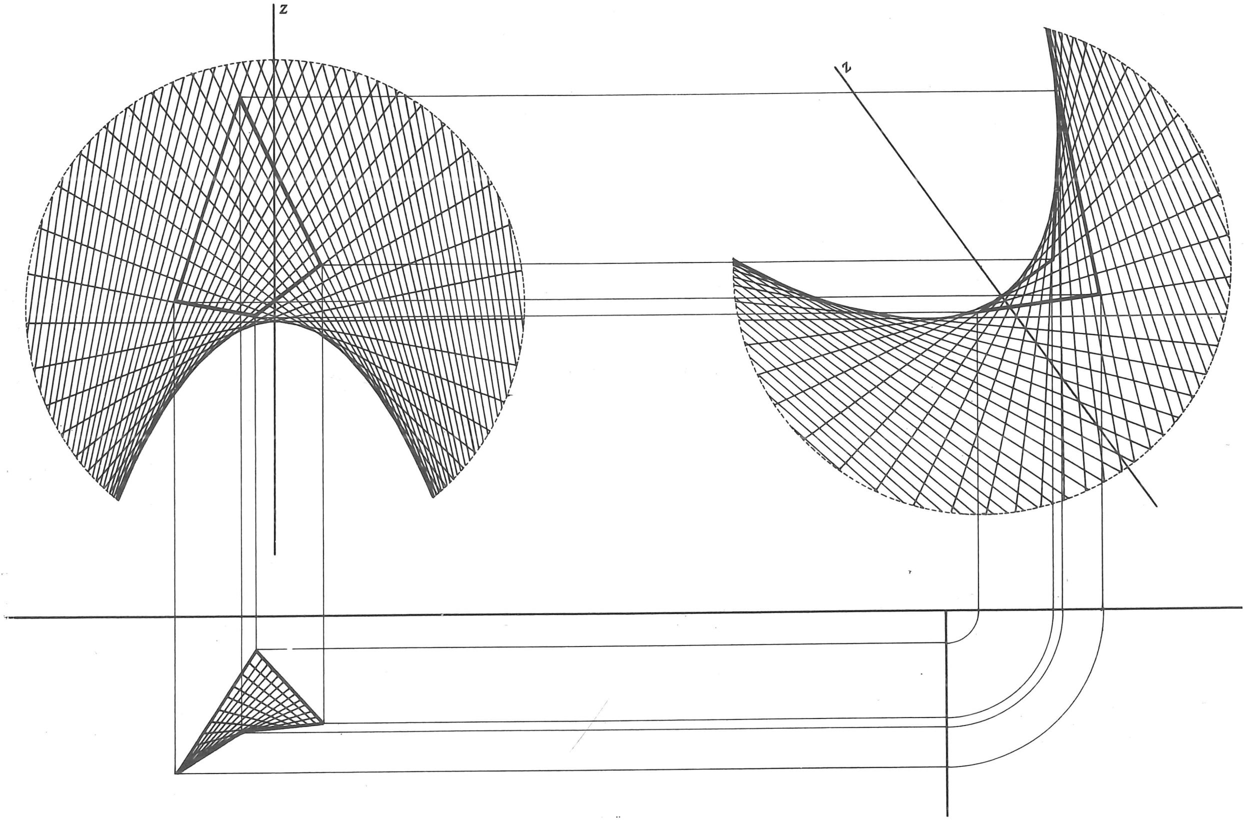


Figure 3—Second Position

a Hyperbolic Paraboloid may have, related to orthogonal planes of reference.

In an attempt to describe these typical positions, each of the four accompanying figures presents the vertical, horizontal, and third projections of a Hyperbolic Paraboloid containing a given unit. For clarity, in the figures the boundless surface of the Hyperbolic Paraboloid has been limited by a circular dotted line. In each figure, the vertical projection at upper left shows the front view of a Hyperbolic Paraboloid with a *unit* limited by four straight line *generatrices.* The horizontal projection, at lower left, shows only the plan of the unit with its two sets of *generatrices.* The third projection, at the right, shows the lateral view of the Hyperbolic Paraboloid containing the same unit.

FIRST POSITION: Figure 2. The Hyperbolic Paraboloid has its Z axis parallel to the planes xz and yz. Under this condition each set of *generatrices* is horizontally projected as *parallel lines.*

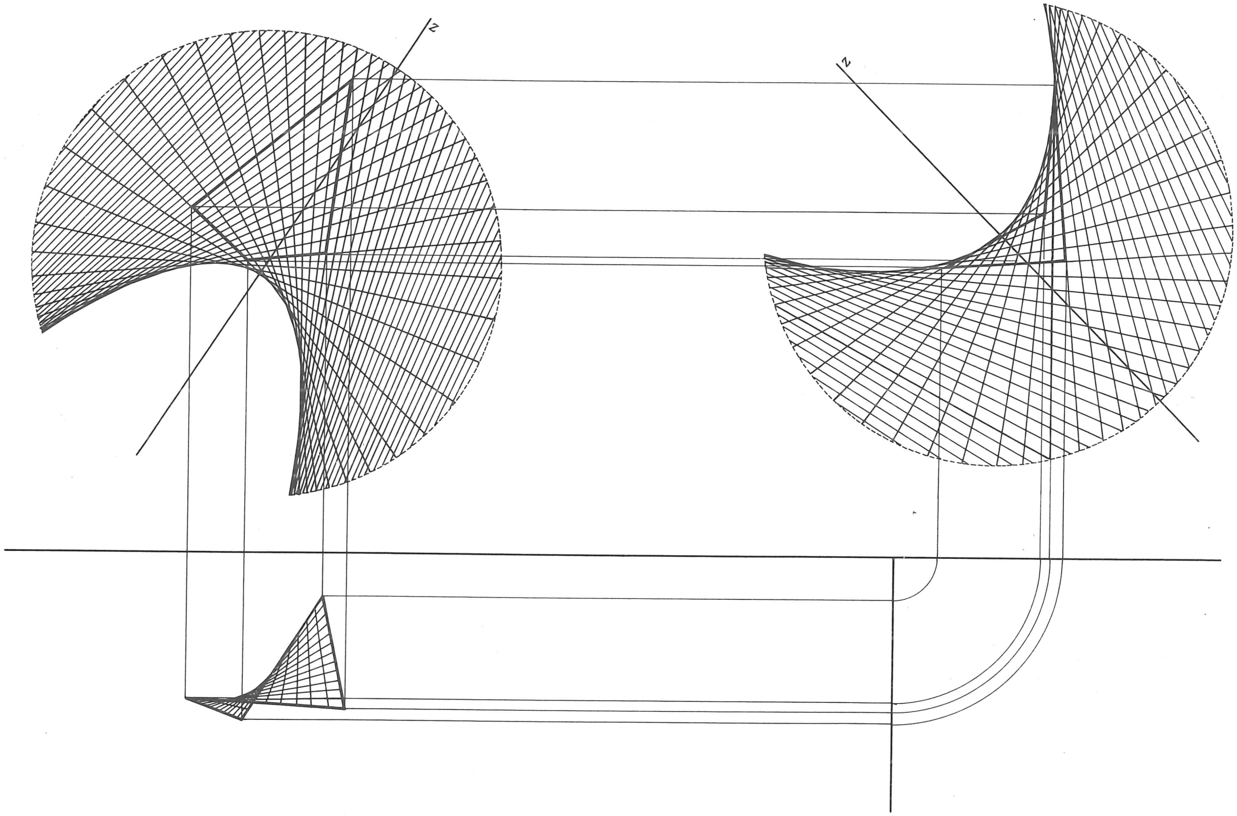


Figure 5—Third Position—B.

This degree of verticality of the Z axis has paramount importance in the determination of the values of the internal stresses developed within each unit. Structurally speaking, a Hyperbolic Paraboloid performs more efficiently with its Z axis parallel to the forces impinging on it, which are fundamentally vertical ones.

The reader, by observing most of the plans of the following plates, can easily determine, through the varied degrees of parallelism among the *generatrices* of each set, how inclined the Hyperbolic Paraboloid to which the unit belongs, is. Through awareness of such inclination, we can determine how close the unit is, to the ideal position in space, described in the First Position.

Structures designed by combining units belonging to Hyperbolic Paraboloids with Second or Third Positions in space have not been frequently used, perhaps because a more complex stress analysis than the one required for the First Position is involved; and also because Hyperbolic Paraboloids have not yet been studied long enough to discover all their structural, three-dimensional, and architectural possibilities.

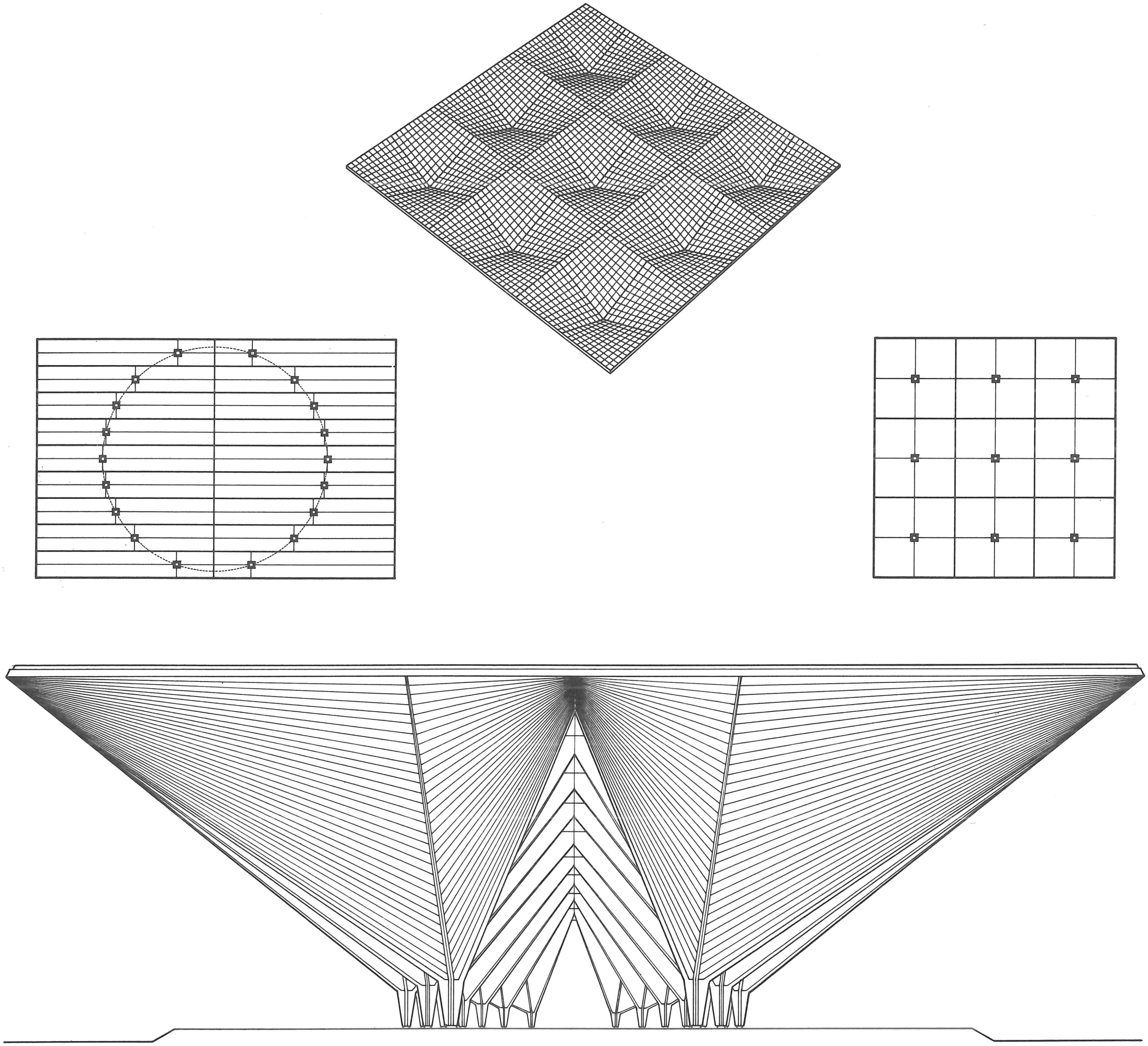


PLATE 2. *Upper: Plan and view of a group of nine symmetric structural units based upon the repetitive use of the structure shown on* PLATE 1, *lower left. Lower: Plan and view of a group of eighteen structural units similar to the ones described above but with their supports displaced in order to be grouped around a circular plan. Although in this case the units are asymmetric, the resultant combination of them originates a symmetric structure.*

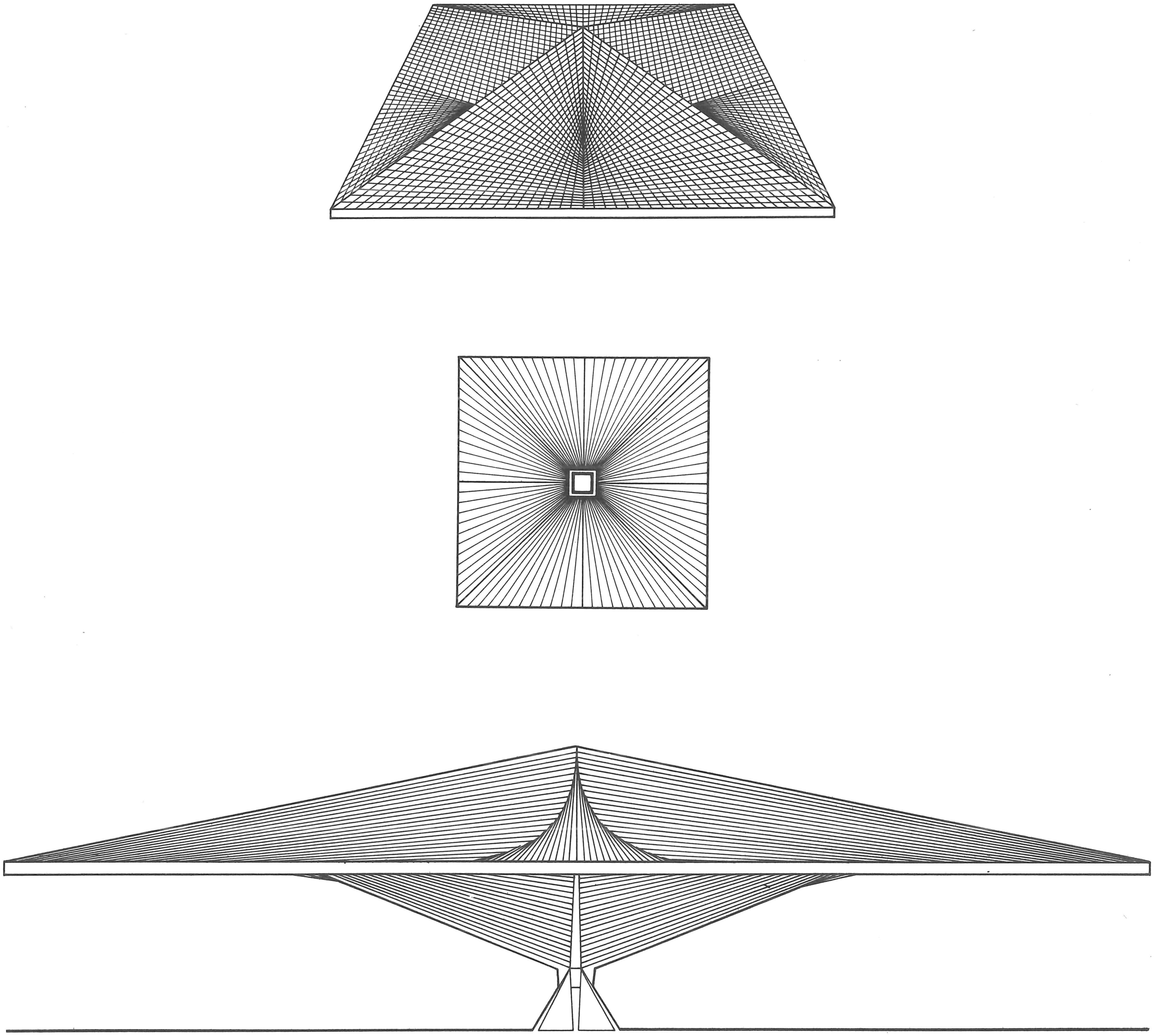


PLATE 4. *Structure originated by combining eight units around a central support. Half of the adjacent edges of the units slope upward and half slope downward, toward the support. It combines in a single form the characteristics of two of the typical structures described in* PLATE 1, *left.*

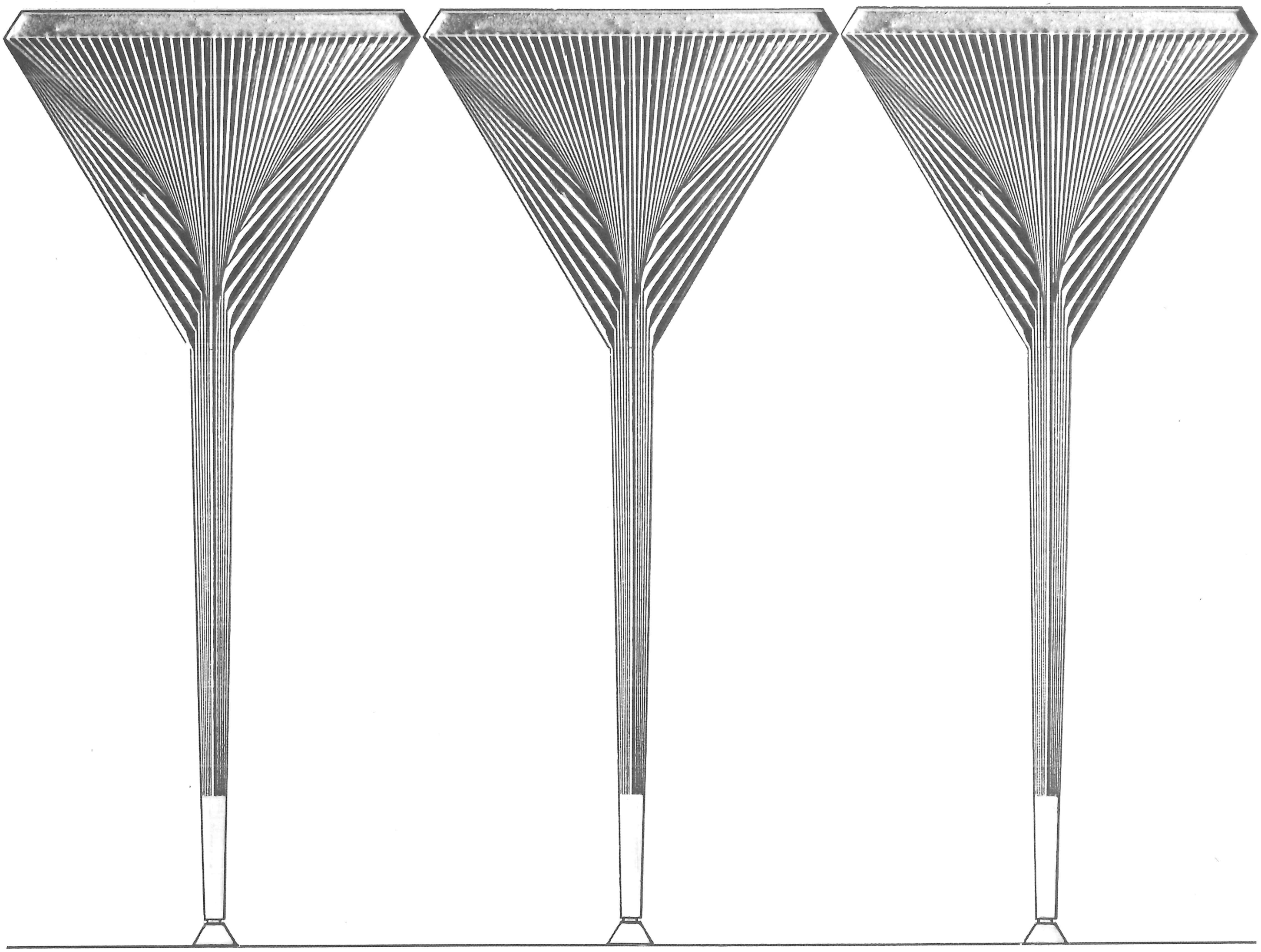


PLATE 6. *Structure based upon the same combination of units as the one shown on* PLATE 5, *but designed using different angles and dimensions — for all its components.*

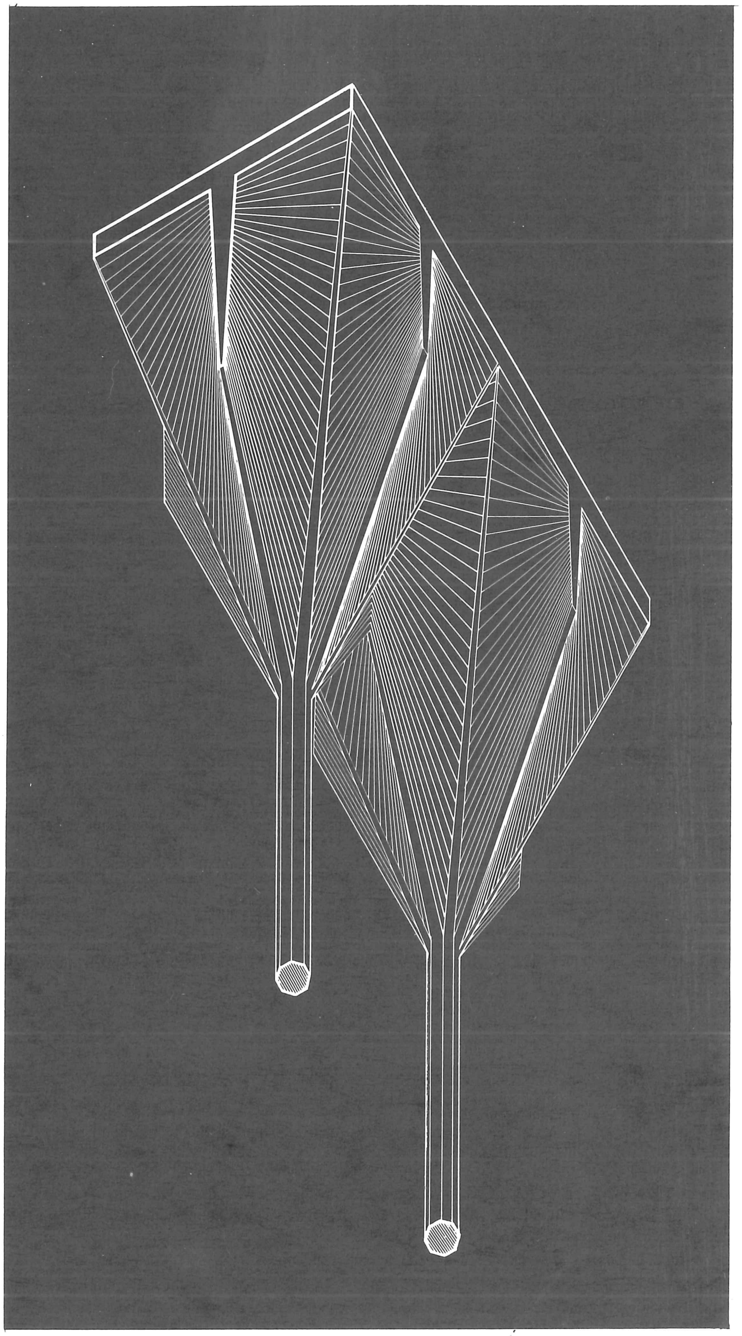
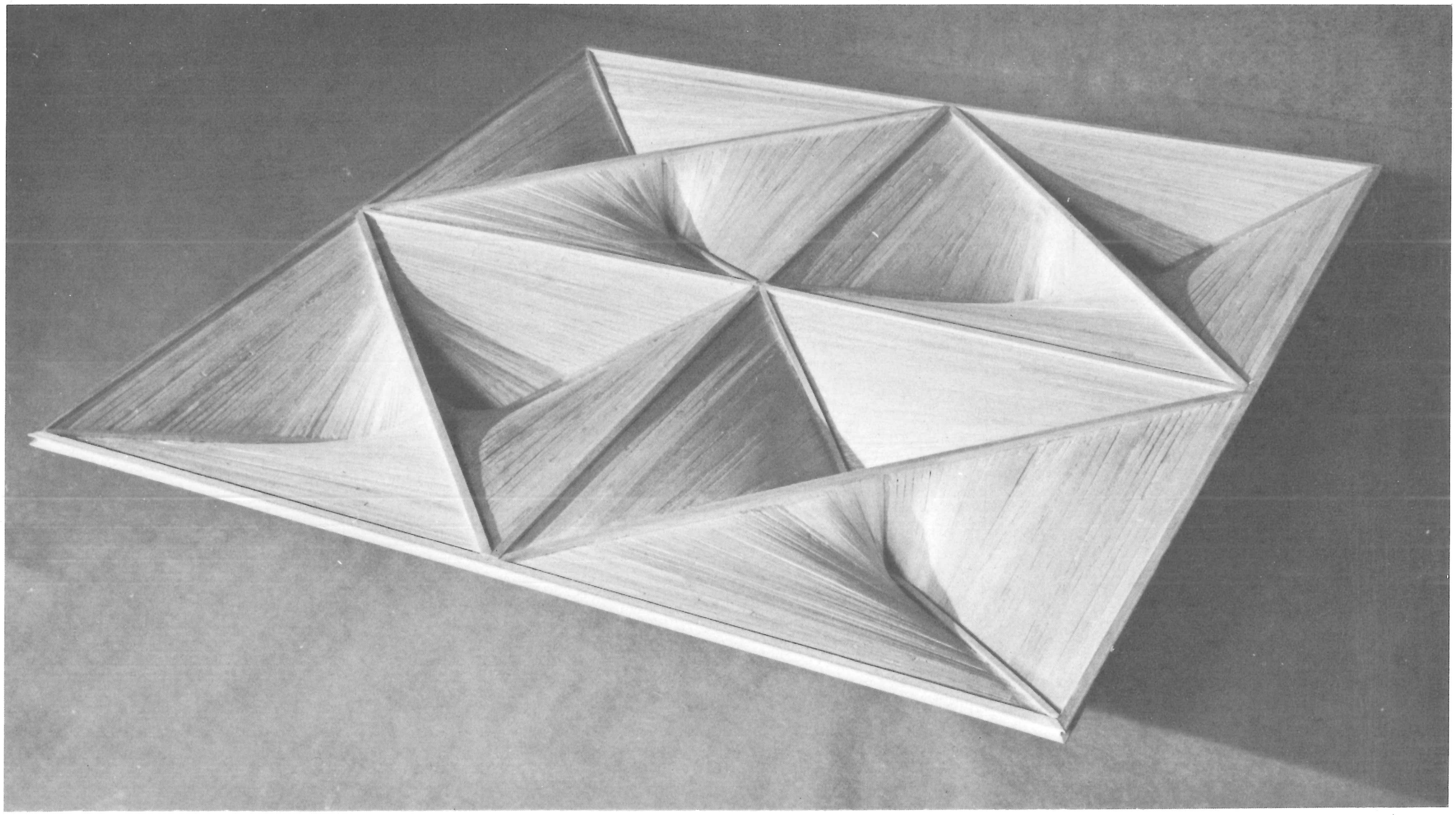


PLATE 8. *Axometric perspective of two structural units similar to the one shown on* PLATE 7. *Each structural unit is joined to the adjacent ones by means of its horizontal and vertical edges.*



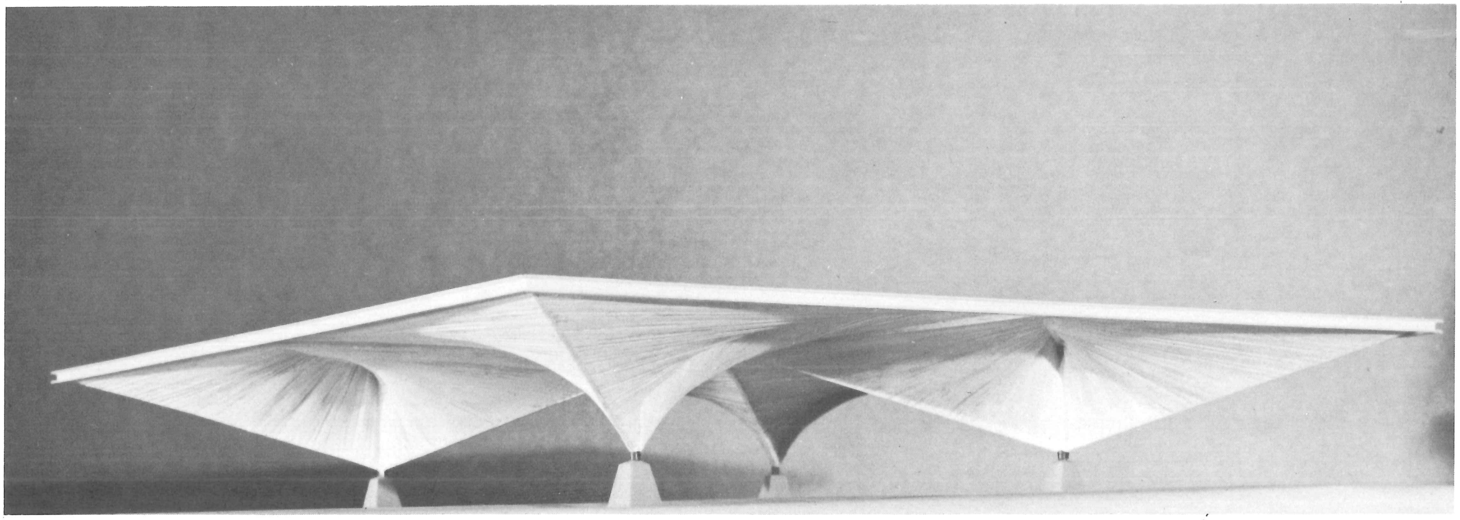


PLATE 10. *Two views of the structure described on* PLATE 9. *A different visual quality can be obtained by rotating each structural unit ninety degrees, around its support.*

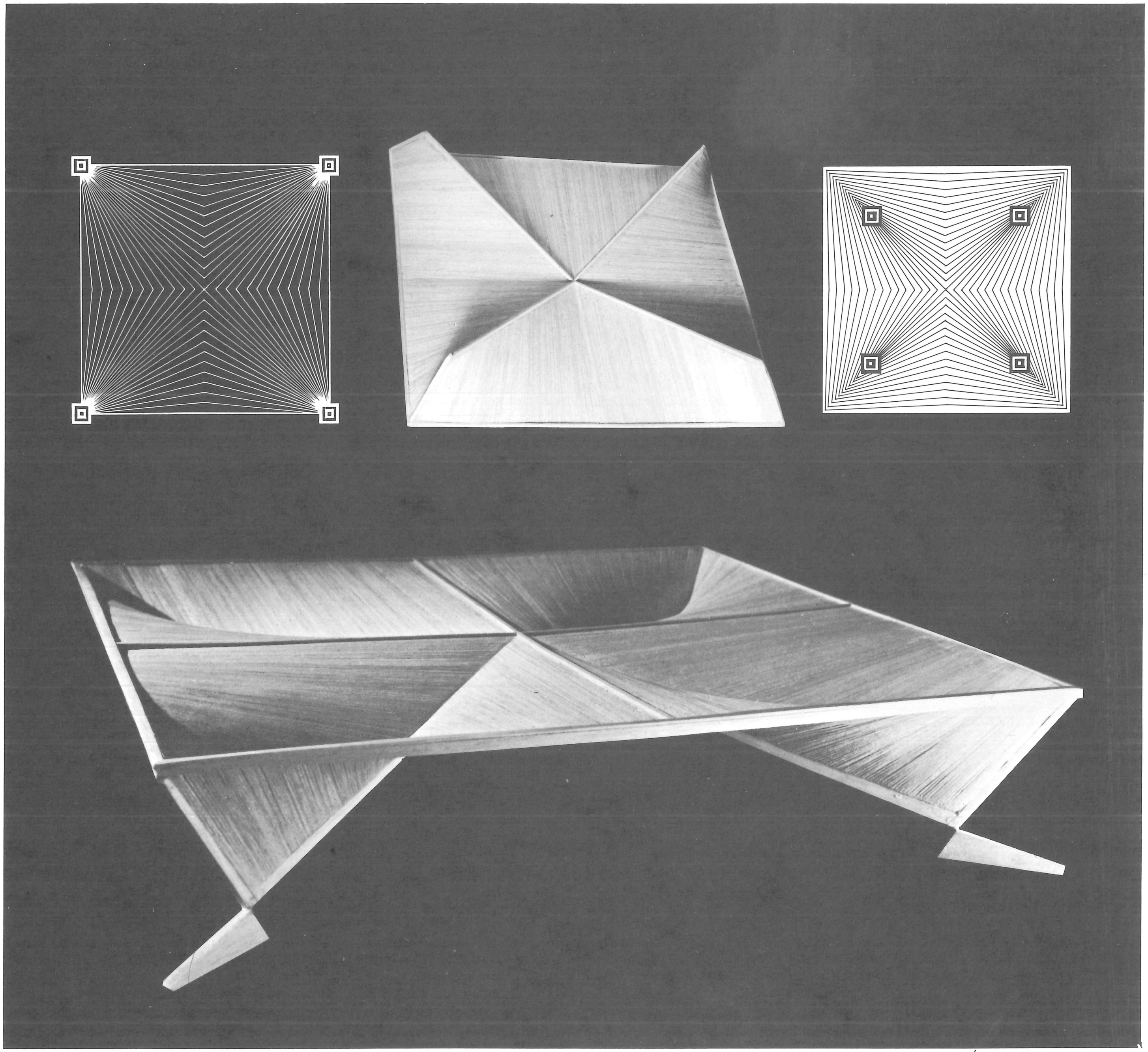


PLATE 12. *Plans and views of a structure originated by combining eight units resting on four supports. Such supports can be placed recessed from the horizontal straight edge or at the corners, as shown in both plans.*

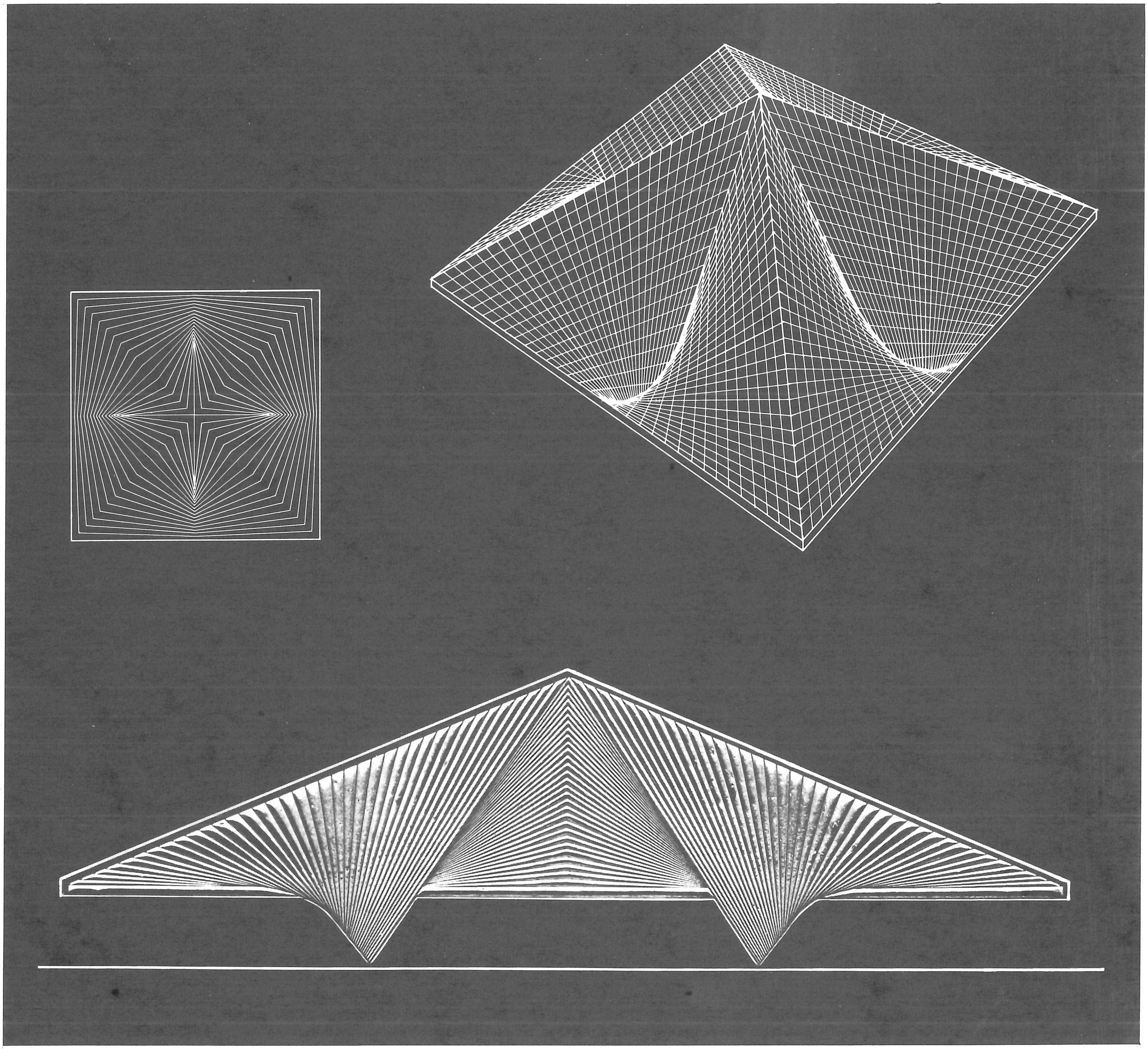
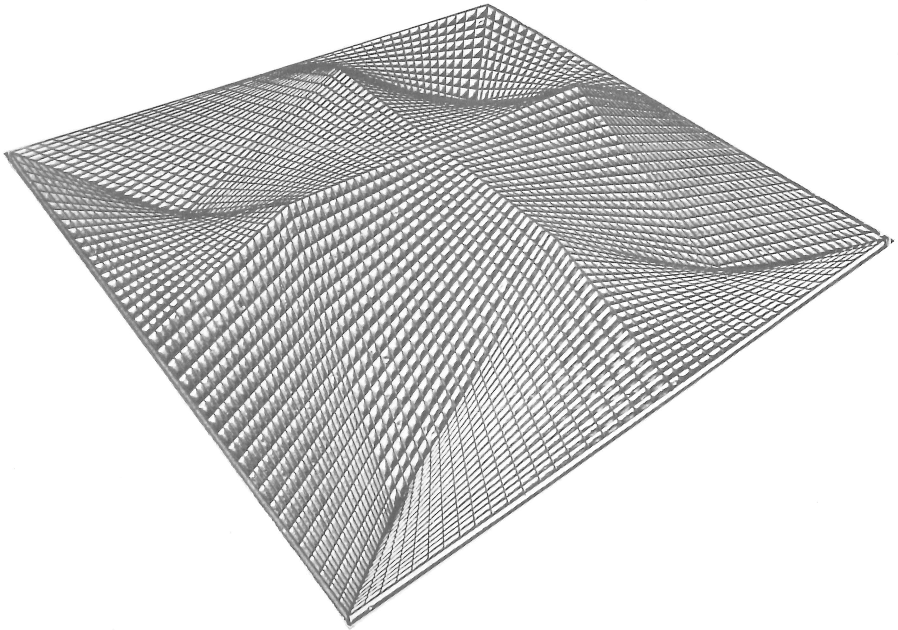


PLATE 14. *Plan, view, and diagonal section of a structure obtained by combining eight units. It differs from the one described on* PLATE 13 *in the position of the four supports, thus combining each set of units within a triangular plan instead of within a square plan as in the former structure.*



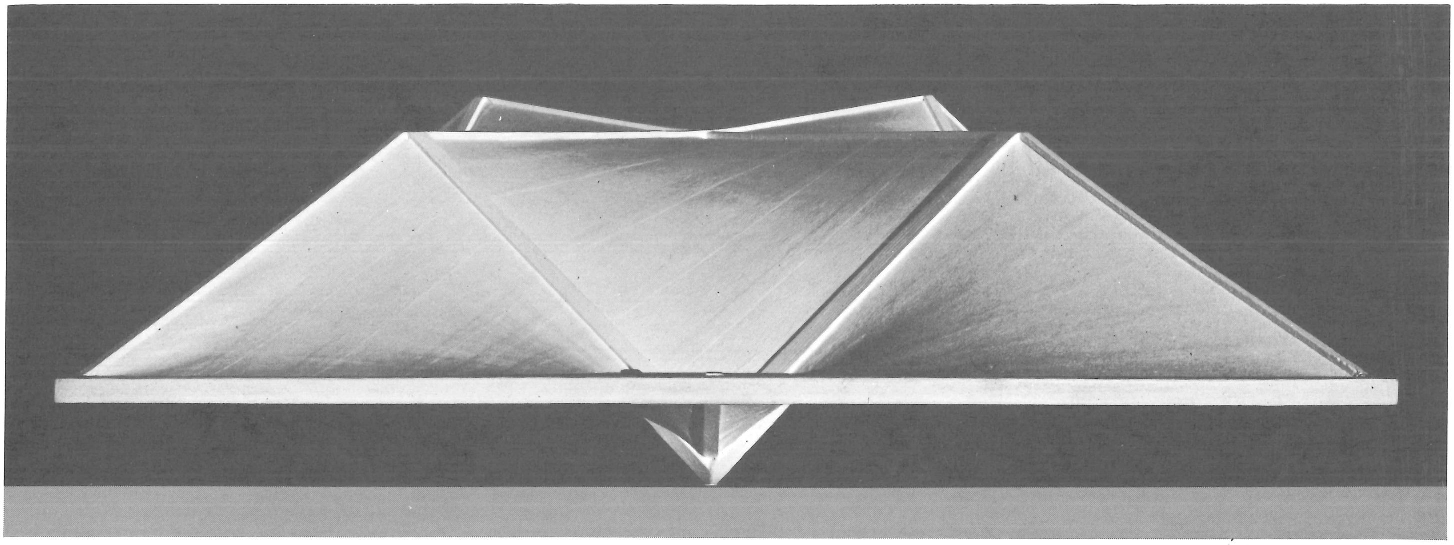


PLATE 16. *Upper: Photograph taken from engraving of the illustration shown on* PLATE 15, *upper right. Lower: Elevation of a structure obtained by combining twelve units as described on* PLATE 17. *It is developed within a square plan and with straight horizontal peripheric edges.*

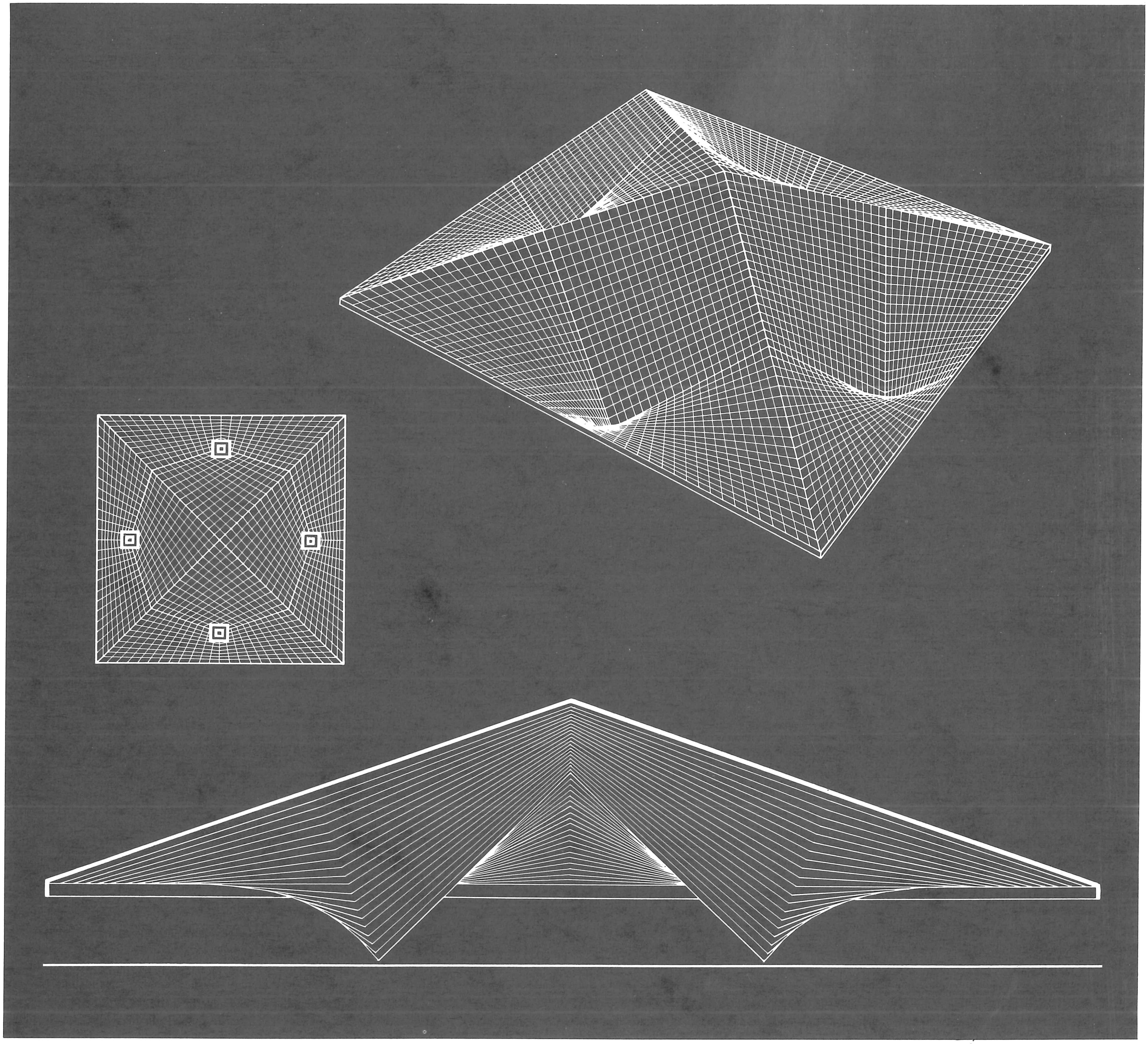


PLATE 18. *Plan, view, and diagonal section of a structure originated by the combination of twelve units, defining a high space at the center and a low one around the periphery. It can be described as a combination of four inclined triangular structural units springing from single supports. It differs from the structure shown on* PLATE 17 *in the slope of the edges of the four central units.*

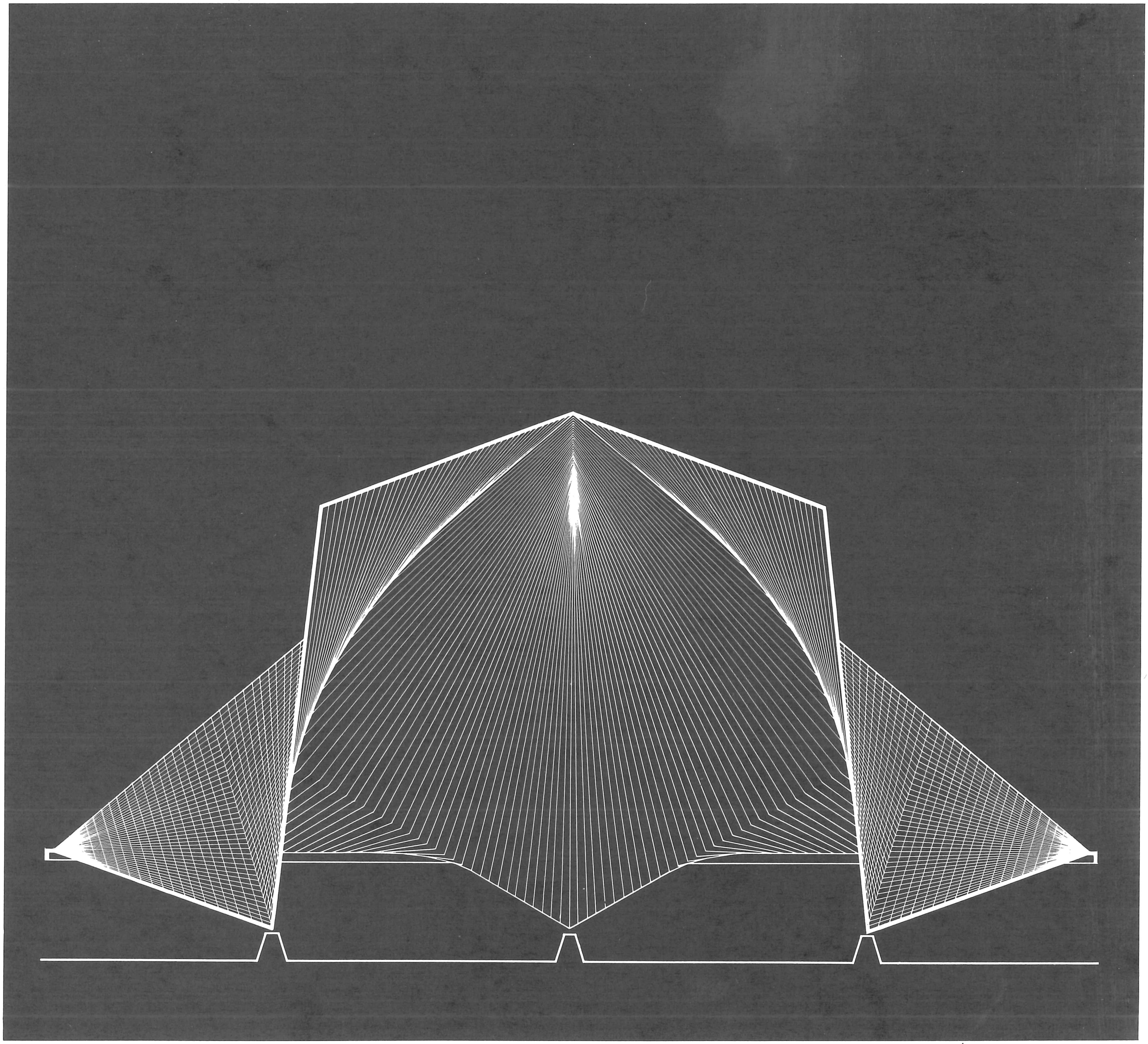


PLATE 20. *Section of a structure based upon the same combination of units as the structure shown on* PLATE 19 *but with an increased vertical dimension for its central space. It shows that a single system can be expressed with different visual qualities by varying the dimension and angles of its component units.*

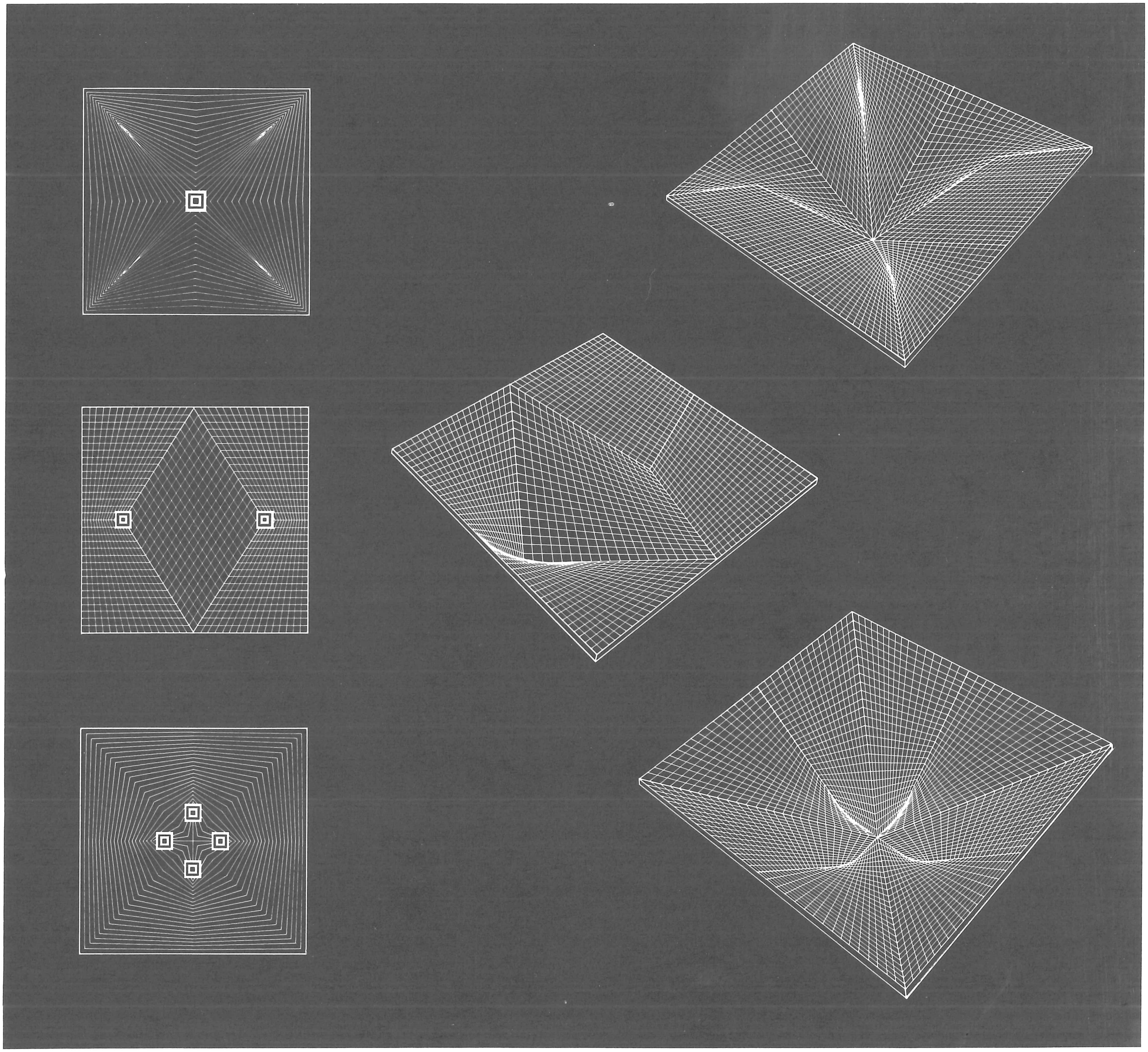


PLATE 22. *Plan and view of three structures with different number of supports. Upper: Structure with eight units around a central column. Middle: Central symmetric unit surrounded by four equal trapezoidal units originates a structure resting on two supports. Lower: Structure originated by the combination of eight equal units resting on four supports.*

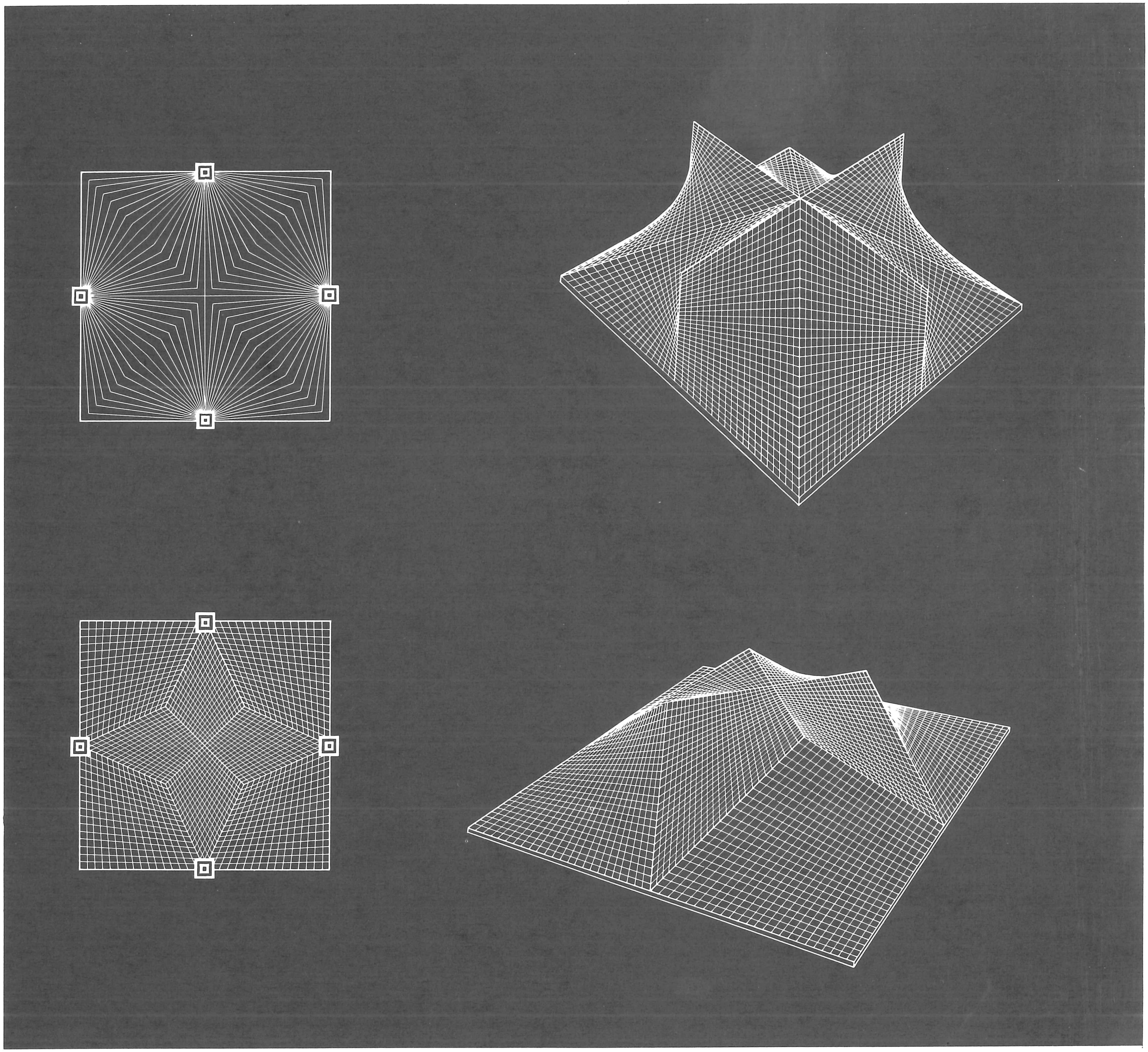


PLATE 24. *Plans and views of two structures obtained by combining eight units and resting on four supports. The central units of the structure of the bottom are combined as in the typical example indicated on* PLATE 1, *upper right.*

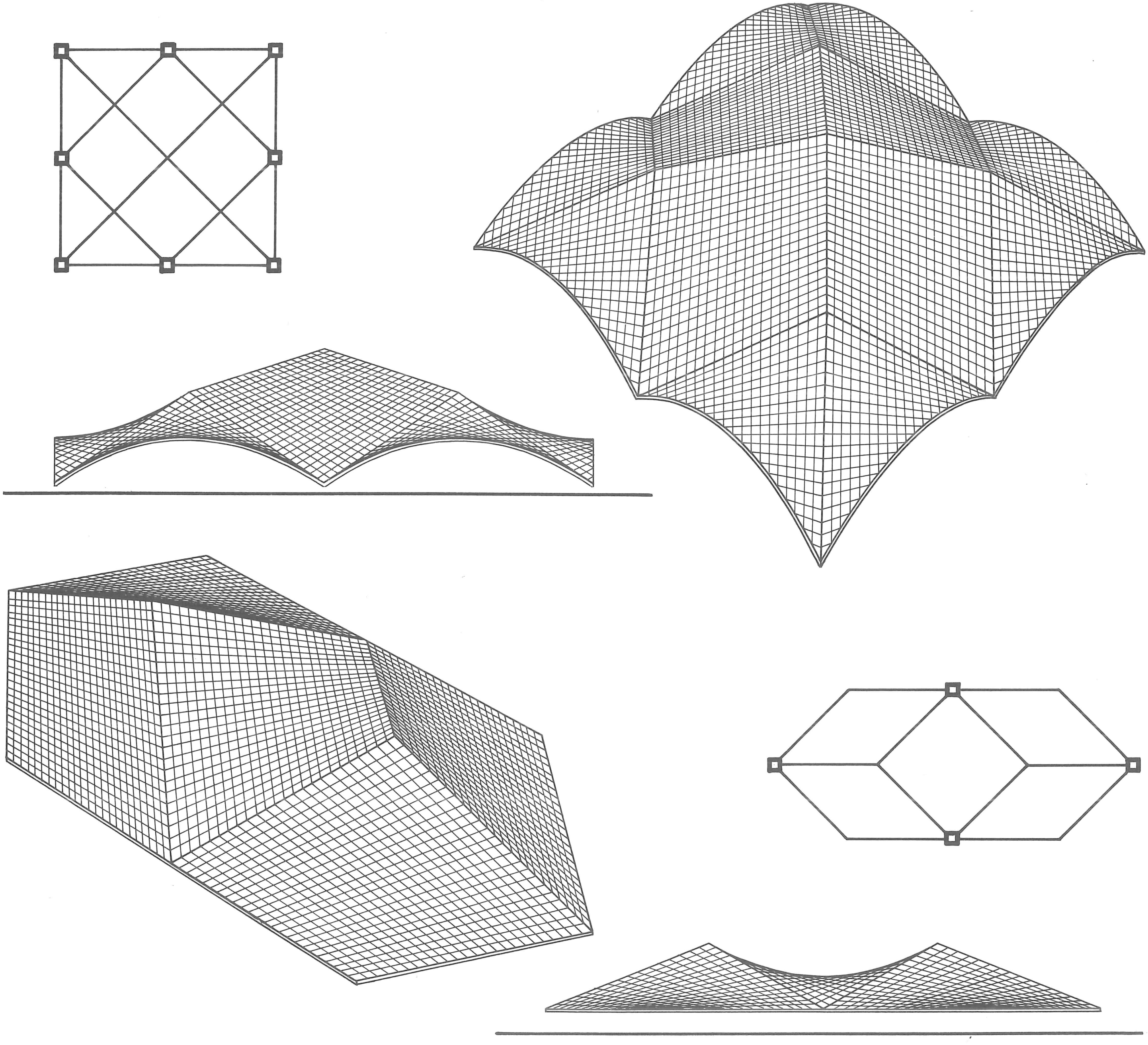


PLATE 26. *Upper: Plans, views, and elevation of a structure obtained by combining twelve units, resting on eight supports which are joined by parabolic arches. Lower: Plan, view, and elevation of a structure obtained by combining five units resting on four supports, and similar to the one described on* PLATE 25.

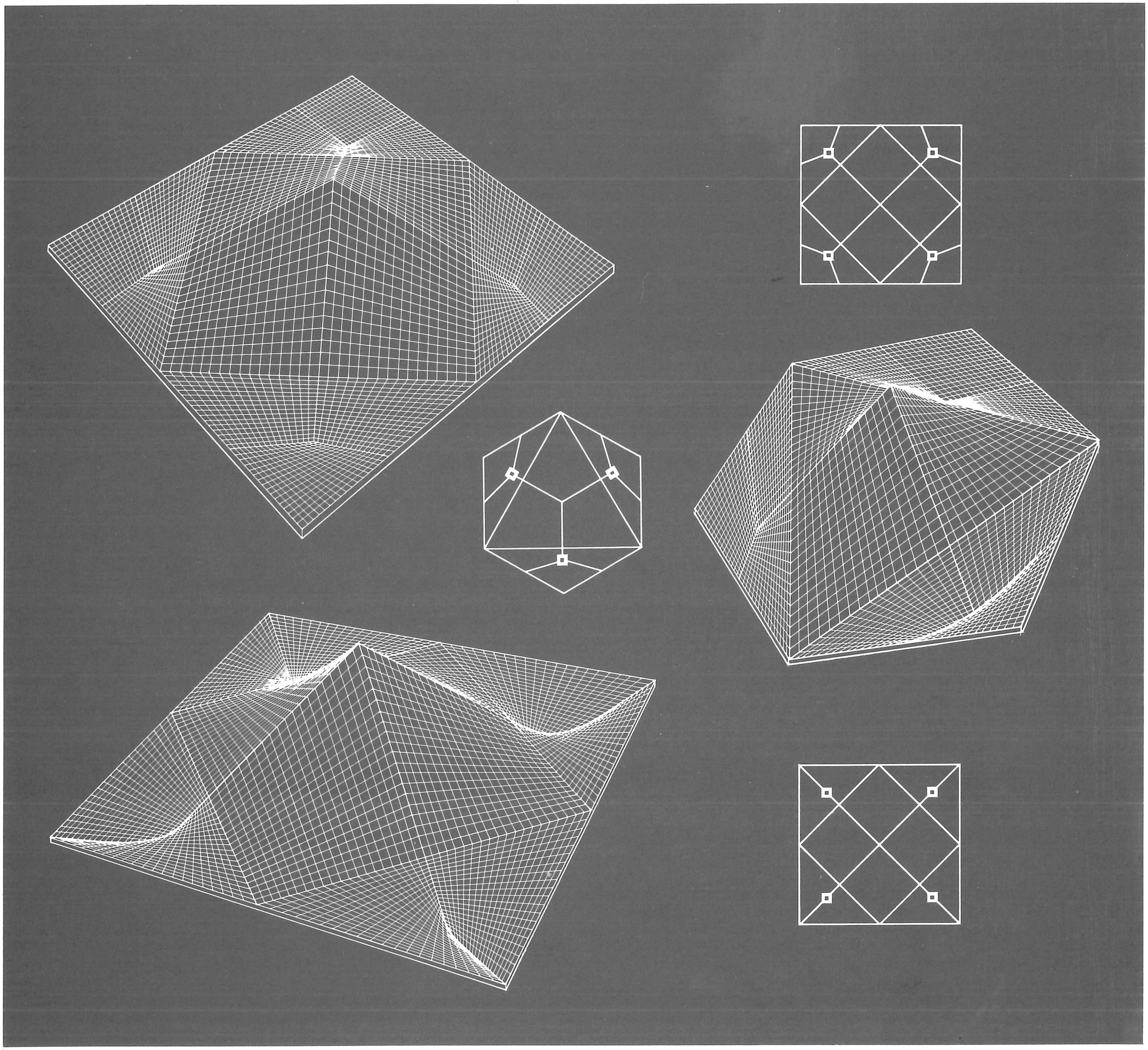


PLATE 28. *View of three structures formed by central units defining a high space and peripheric units combined within a triangular plan. The four central units are combined as in the typical example, upper left, shown on* PLATE 1.

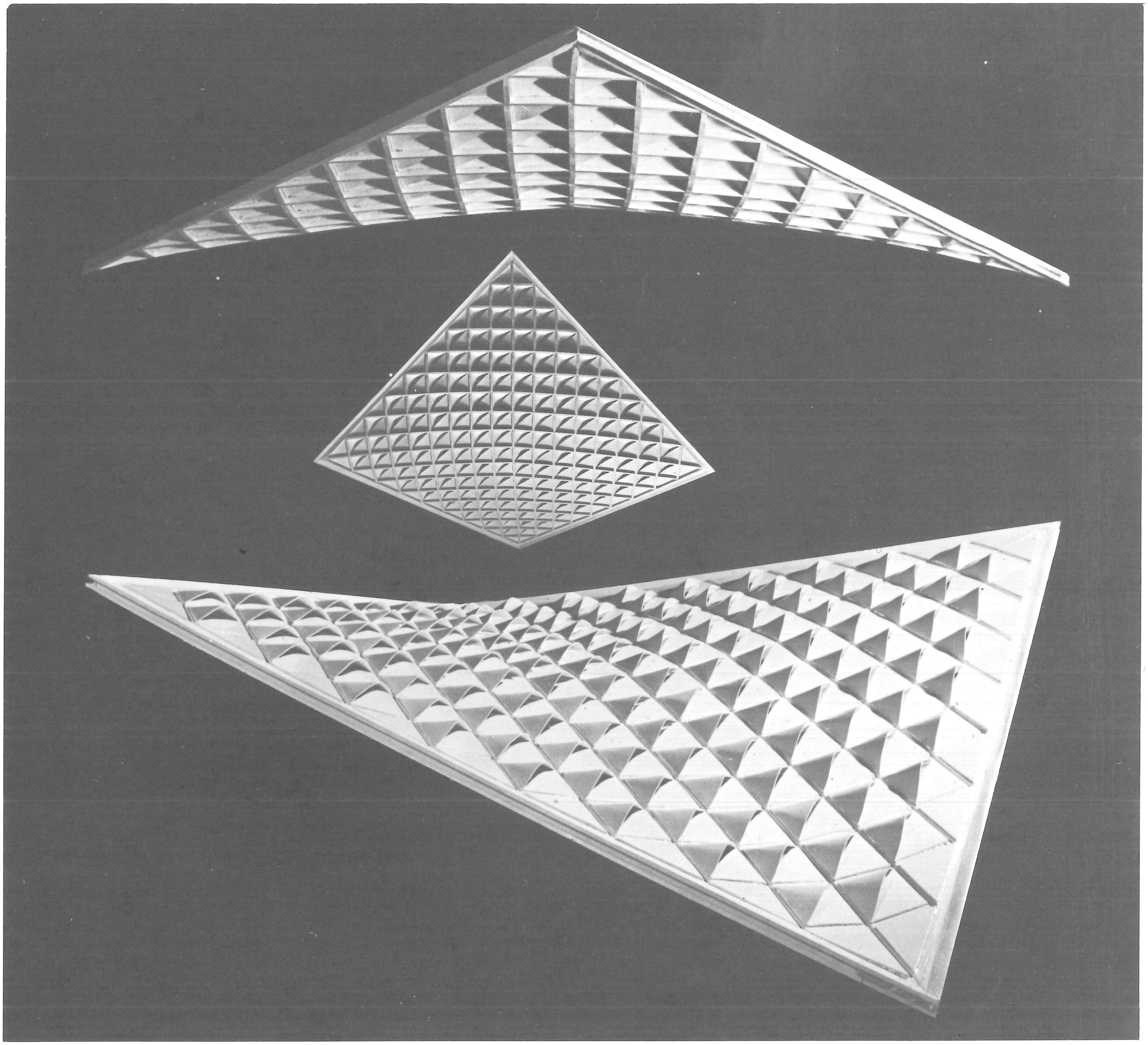


PLATE 30. *Three views of a structure based upon the use of a single unit resting on two supports. Upper: Diagonal view showing that the surface is defined by a lattice work of parabolic arches. Center: Underneath view showing that such arches follow the diagonal direction determined by the opposite vertexes of the unit. Lower: Side view showing the use of thin conical fillers to complete the surface, and to allow the pass of light.*