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Cover: Photo taken from the Temple of the Holy Family designed by Antonio Gaudi, Barcelona, Spain 1910 (from El Arte de Gaudi)

## VOLUME 6 NUMBER 1

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by Felix Cardellach, Engineer and Architect*

The speculative regions, or the ideal orders, are saturated in an atmosphere of unheard of simplicity; surfaces float isolated in the space without need of physical existance; the medium does not offer resistance, the matter of the bodies is completely isotropic. If some variations exist, they are always due to a known cause; no unknown influences, no unforeseen accidents occur.

In this ideal region, man discusses and rationalizes using the privilege of his imagination, with absolute rigor (sternness) with eternal truth-all statements have a complete certainty in this region without darkness, perpetually lighted by some isolated beam of the "Great Divine Science".

This is the "Parnassus of Mathematics", whose rationalism is skillfully synthesized in geometrical concepts and analytical expressions, something like a spiritual gear of rational deductions that carry us swiftly into the discovery of new truths.

But the application of such prodigious mechanism to the material world gives us results that are often imperfect, due to the tremendous complication of all the facts that shape life-whose laws and materials are unknown-in its rhythms and intimate properties.

Nevertheless, this application has been achieved by the engineer, who has simplified the complexity of natural phenomena for this purpose with conventional hypotheses. It is an ingenious bridge between reality and the speculative spheres where mathematics reign.

Following this method, conclusions are stated; after that we proceed to interpret them in practical situations. But it happens that, since we have started with a hypothesis, it is not always possible to find a desirable concordance between theory and practice, and we become disillusioned. In fact, very often the mathematical world is useless for technology. The preceeding conclusion is also true in the art of construction.

Today, the major mental efforts of the builder lie in the fantastic discovery of the law of reaction of materials against a system of external forces.

This is the philosophical base of what is called strength of materials, which looks like pure science since it is covered with an analytical coat. Strength of materials, like any science, starts from the material world with hypothesis, evolves in the mathematical regions and comes back to our practical cases where we interpret the last aspects of the metamorphasis that has transformed the initial formula.

The newcomers in science, the imaginations not well educated, are not aware of this singular cycle proper in any technique. They believe in "real facts" and only search into the algebraic transformations for the exact solutions of natural phenomena.

[^0]Why not avoid, if possible, this confused and painful round-about in the problems of construction, by educating our senses in direct observations and reasoning? One of the ways to achieve this is to establish a methodical classification of structures and exercise a synthetic observation of different mechanical behavior of structures.

Such kind of studies can be adopted with successful results. Actually, it is the same method used in studying Plastic Arts (Fine Arts). In History of Art, the analysis of determinant causes, its evolution and general principles develops a sensibility that enhances the spirit in the work of conception and creation.

It is possible to follow an analogous path in teaching mechanical-constructive problems; analyzing for that purpose the intuitive engineering developed in the past, forerunner of our actual structures. At the same time, will be required an intense mechanical sensibility, much more developed and fructiferous than the one that we have now, through the immediate analysis of constructive forms whose creation is the product of centuries.
The mind of future builders will be encouraged in the conception of new structural forms required by our civilization, a problem that nobody has been taught to solve. In addition, the complete dissassociation that constitutes the knowledge of building technique, relegated today to the simple accumulation of unrelated problems will disappear by virtue of magic synthesis.

Is there in the conglomerate of facts without apparent relation that constitutes the origin of constructive forms and its method of verification some common essence, a general law such as is found in other fields, that will completely regulate all phenomena? From the discovery or approximation of this law, perhaps we can achieve some integrated method that will facilitate the verification of stability in all constructions, and provide furthermore a light in the search of new and infinite structural forms that exist in the region where, due to the expert hand of seer mechanics, had painfully sprouted the lintel, the arch, the tensor, the cantilever.

Let us think, then, about the complex nature and function of form-resistance in construction; let us remelt in the same crucible the multiple and abundant series of methods of verification that engineering offers; let us analyze the evolution, influences and relationships of the different types of structures, old and new. We will find certainly settled in the bottom of this interesting analysis, a true synthesis, a positive origin of practical advantages where we can inspire our approach to constructional problems.

roof of Casa Batllo, Barcelona



## THE IMPORTANCE OF CONSTRUCTION TECHNIQUES

## by Pier Luigi Nervi

designer, engineer, builder and
Professor of Engineering, University of Rome

I wish to emphasize that these conclusions stem from the fact that from the beginning of my professional activity, I have been both designer and contractor. This coincidence has allowed me to consider the field of structures from two viewpoints, which for the most part are kept artifically separate.

This lucky coincidence has enriched both my activity as designer and as builder, and proved to me that progress in the field of reinforced concrete is impossible unless the architectural and structural aspects of a problem are studied by the same person and, at the same time, by collaborating persons of different skills.

The marvelous architectural and structural possibilities of reinforced concrete are only too often suffocated by the constructional deficiencies of the traditional building methods and by a lack of careful organization.

The need of a temporary structure, the scaffolding, which gives form to the concrete and supports the reinforcement, complicates the technical problems and makes more acute the economical problems of reinforced concrete, as compared to those of steel or masonry.

It is as a builder that I have been able to appreciate in all their importance the economic aspects and the technical difficulties of building in reinforced concrete.

I first became aware of these problems in 1929 when I designed and built the Florence Sports Stadium.

Confronted with the high cost of the scaffolding for the covered stands and with the complicated forms for the elicoidal stairs, I realized that one should not forget construction during the initial phase of a design.

The influence of the construction techniques on cost became even more apparent to me when I first tackled the problem of the two enormous hangars put under bid by the Italian Air Force in 1935


Hangar in Italy (1935). Span: $333 \times 133$ feet in reinforced
concrete.


[^1]I wish to point out to you that these hangars, as well as the Florence Stadium and almost all of my other designs and realizations, were studied by me with a definite bid in mind. In these bids a number of chosen designer-contractors were invited to present a project and to submit a bid on the basis of general schemes studied by the client.

I was so taken by the complex structural problem presented by the hangars that I actually did not study with enough care the construction problems involved. But when I began building these structures I realized full well that the forms were extremely costly, that a lot of timber was wasted, that labor was not used too efficiently, and that from then on I had better find construction techniques well adapted to the problem at hand if I was to beat my competitors and make full use of the inexhaustible potentialities of reinforced concrete.

In 1939 I was given the opportunity to again study this type of hangar and to put to use the lesson I had learned in 1935.

Economy was this time even more important in view of the total amount of work to be done: 6 hangars, $300 \times 120$ feet, 36 feet high. I had to find a cheaper solution than the one I devised in 1935.

My first thought was to simplify the structure by making it symmetrical with respect to its two principal axes and by sustaining it on only 6 buttresses. My intuition told me that this symmetrical structure should work better and that the elimination of a large number of columns in the back and on the sides of the hangar should represent a substantial saving. Moreover, symmetry would allow a more accurate mathematical analysis and hence an increased economy in design. The mathematical results of the analysis were checked by model analysis, as I had done for the 1935 hangars.

But the two essential points were to reduce the dead load and to simplify the formwork. Among the various possible solutions I felt that the most promising would consist in prefabricating all the elements of a network of shell ribs. Prefabrication would allow the construction of very light ribs, and I decided on a triangular cross-section for these elements. The four ribs meeting at a joint of the network were to be connected by welding their reinforcing bars and by filling the joint with high strength concrete.

The validity of all these assumptions was tested by means of carefully planned experiments.

The economic advantages of the new system led my company to win the bid and to build the hangars with excellent technical results.

You may notice from the photos that the different construction techniques used in building the two types of hangers have substantially influenced their architectural appearance.

But full light was really thrown on the importance of construction in connection with the architectural solution of a problem by my study for the roof of the great Exhibition Hall in Turin. The hall was to be $300 \times 345$ feet and to have a half-dome of 120 feet diameter at the back. Its roofing problem was anything but easy.

In order to obtain a solution both harmonious and impressive the following situation had to be faced: a very limited amount of time was available to build the hall; a low bid was necessary to obtain the job (other firms had been invited to submit designs and bids); the architectural, acoustical and the illumination aspects of the solution were all important.

Without mentioning the many solutions I considered from time to time, I would like to make clear that the adopted solution, based upon the use of large, undulated arches, and which I contemplated quite early since it satisfied both esthetic and functional requirements, would never have abandoned the field of pure speculation had it not been for the simultaneous consideration of a construction method well adapted to its realization. In fact, it was quite clear that the structure could not be built economically by means of the usual scaffolding and formwork or by means of any of the other usual construction procedures.

I therefore decided to exploit the potentialities of ferrocemento, which I had invented and tested in previous work and more particularly in the construction of ships of modest dimensions. I was thus able to solve the problem with ease by dividing each undulation into separate elements about 12 feet long and weighing about $3,300 \mathrm{lbs}$. These elements were to be prefabricated and would act as connecting links between the reinforced concrete arches to be poured at the top and bottom of the undulations.


Detail of Turin Exposition Hall.

This scheme permitted the prefabrication of the elements while the foundations and the other structural elements of the work were being poured. Each element was stiffened at the ends by two diaphragms, which, when set next to the diaphragms of the adjoining element and connected by $21 / 4$ inches of mortar, made the juncture of the elements easy and strong.

The pouring of these elements took place without difficulty and without requiring double forms (as would have been the case if regular concrete had been used). The high percentage of cement ( $1,600 \mathrm{lbs}$. of cement to 30 cubic feet of sand) allowed the curing of these elements in two or three days (depending upon the room temperature); they were then piled one on top of the other.

The elements were lifted in place and approximately 2,700 sq . ft . of roof were built each working day. The roof was built in three sections in order to re-use the moving scaffolding. The connection between the undulations and the supporting buttresses, which were 23 feet apart, was obtained by means of prefabricated "fan" elements of ferro-cemento and of regular reinforced concrete elements.

Large domes may be easily built by means of prefabricated undulated elements of ferro-cemento. My design for the great Sport Palace in Vienna with its dome of over 455 ft . in diameter is based on this idea. This project was studied in collaboration with my son Antonio, who is an architect, and was submitted to the competition set up by the city of Vienna, but unfortunately nothing came out of it. I still believe it worth while to describe some of the aspects of this solution.

Apart from its architectural value, which may not be objectively measurable, there is no question about the project presenting an efficient solution from the statical, the economical and the constructional viewpoints. Moreover, insulation, ventilation, illumination and sound absorption were all considered and found logical and simple solutions in the system adopted. I have often noticed that a good structural solution almost automatically satisfies all the other requirements of a building.

The half-dome of the Exhibition Hall in Turin was built by means of a structural system I had previously studied, and

applied to more modest structures soon after the end of the war. This system also was suggested by the necessity of labor and saving timber.

The system is well adapted to the construction of both domes and barrels and consists in dividing the surface to be built into elements from 18 to $26 \mathrm{sq} . \mathrm{ft}$. in area and in prefabricating these elements on cement forms. These forms are built on a wooden form which reproduces one of the identical shell portions.

Each element has edges which create a trough about 3 in . wide between two adjoining elements. The reinforcement is laid in these troughs which are then filled with concrete and become the stiffeners of the completed structural system. The elements of t.ee half-dome in Turin are made out of ferro-cemento and are about 0.8 in . thick. A movable scaffolding is used for their erection, but no actual formwork is needed. The shape of the elements can be freely chosen and since the stiffening ribs are in sight, the result is often most interesting from an esthetic viewpoint.

A year after the completion of the large Exhibition Hall in Turin, my company was asked to submit a design and an estimate for a roof of approximately $168 \times 197 \mathrm{ft}$. to cover a hall adjoining the main hall.

In this case, also, the time available for the construction of the structure was very limited, since work could not start before November and the structure had to be completed by the following March.


Italian State Tobacco Factory (1949). Movable precast concrete forms.

Prefabrication became imperative. The basement of the hall already built lent itself ideally to the prefabrication of the elements during the Winter months.

I studied a groined dome supported by 4 inclined arches lying in the plane of the dome reaction. The dome was built by means of elements defined by horizontal lines and by lines parallel to the corners of the dome. The lower part of the dome was ribbed but not filled so as to have a band of light at the foot of the dome.

The perimetral floor of the hall, 30 ft . in span, was to be built by means of prefabricated, undulated elements of ferrocemento, set one next to the other and joined by a light slab poured on top of them. This type of construction gave a pleasant undulated appearance to the ceiling and permitted good acoustical insulation.

The floor was put up in a few days after the undulated beams had been poured during the months of January and February. The beams are 0.8 in . thick except for the lower part of the undulation where the thickness grows to 1.56 in . to be able to cover the reinforcement properly.

The beams were poured on forms of cement, which were prepared on special gypsum forms. The lower surface of the beam which was in contact with the forms, was perfectly smooth, in fact smoother than the finished surfaces of most reinforced concrete structures.

This structural system can be applied to a variety of curved surfaces. I have used it repeatedly to build domes and cantilevered structures.

The same system led to the realization of the interesting roof of the large hall of the New Terme di Chianciano. The eliptic shape of this hall, $80 \times 92 \mathrm{ft}$., required the preparation of forms for half the elements of the roof. Nevertheless, the solution was economical and pleasing and allowed the erection of the roof in record time.

On the occasion of the bid for the new warehouse of the Italian State Tobacco Manufacturing plant in Bologna, I studied a system of movable forms of ferro-cemento which opened up new structural and esthetic possibilities by allowing the pouring of beams without the limitations imposed by wooden forms.

This newly acquired freedom not only allowed the most logical choice of cross-section and of location of the main and the secondary beams, but brought about the construction of reinforced concrete floors in which the beams follow the lines of principal bending moments. This strict adherence to statics brought about a substantial economy of materials.

It is interesting to notice the esthetic appearance of the beam pattern, which evidences once more the mysterious connection between the laws of physics and our esthetic sensibility.

It might finally be of some interest to mention the rapidity and economy of construction obtainable with the use of ferro-cemento forms for the pouring of a structure or for the prefabrication of structural elements.

In March of 1955 the Fiat Company of Turin asked a number of contractors to submit designs and bids for a large industrial building, three stories high, 1,930 feet long and 61 ft . wide, and for 8 additional buildings connected to the main structure, also three stories high, 20 ft . by 20 ft . in plan. Altogether, the building of $450,000 \mathrm{sq}$. ft. of floors was required.

The ground floor had a line of intermediate columns and its slab was built by means of movable forms of ferrocemento. The slabs for the first and second floor were supported on prefabricated beams, 61 ft . long, set $7^{\prime}-6^{\prime \prime}$ apart, and poured on movable forms of ferro-cemento.

The job was given to my company on April 1, 1955. The pouring of the foundations and the preparations of the forms took two months. The floors were started towards the middle of June and completed towards the middle of October. The entire building was ready for occupancy on December 31st of the same year.

I want to call your attention once more to the importance of the constructional process on the economy and on the esthetic appearance of a structure. Not only the general form of the structure, but also its details and the type of finishing obtainable, depend on the construction technique.

The simultaneous solution of our three essential problems -the structural, the constructional and the architectural problem-is the only starting point for the fruitful development of the inexhaustible potentialities of reinforced concrete.

## TIME DRAWING

## by Duncan Stuart

Associate Professor, School of Design

The drawings below are the results of an investigation carried out by a sophomore design class during the 195556 school term. The intent of this investigation was to seek those properties of drawing which would enable a student to discover a more true visualization of the spatial properties of an architectural situation. It became apparent in the early stages of this investigation that a system of drawing which would permit the recording of the time sequence of viewing was essential-that the system of "Italian Perspective" which traditionally has been used for this purpose would be inadequate. A brief consideration of the projective process known as perspective suggested that the job of recognizing the full panoramic properties of any position in space was a fairly straightforward geometric process which, in actuality, turned out to be basically simpler in its conception than the "flat plane" perspective used heretofore.

This development consisted of changing our plane of projection from a flat plane to a cylindrical one. This new system has essentially the same inaccuracies as flat plane perspective, since the distortions along the axis of the cylinder are still present. The main virtue of the system, however, would not lie in this direction, but rather in the direction of the ability to control the continuity of view through the full $360^{\circ}$ sweep of our eye, keeping distortion at a minimum.

The illustrations found on this page show the method by which this construction is made. With reference first to the plan view, we see a configuration of elements on which has been superimposed a series of equally spaced radial straight lines and equally spaced concentric circles. The radial lines and circles are generated by the point on the plan corresponding to the observer's desired position. The radial lines are indicated by number, the concentric circles by letter. From these lines and circles
the development of the cylindrical projection (strip drawing) may be easily devised. First of all, an eye level line is established which projects as a straight line on the developed cylindrical surface. We then mark off equal segments at any suitable scale along this eye level line which correspond to the equiangular radial subdivisions in the plan. Through these points along the eye level we construct a series of verticals. Now with reference to the plan again, we employ the first concentric circle on the plan to determine the scale distance of a single segment of the concentric circles and with these at the cylindrical projection we place a horizontal line



below the eye level in such a way that the proportion of the rectangle created by eye level, a pair of adjacent vertical lines, and the new horizontal line establishes the proper ratio with respect to the eye level. That is to say, if we wish to denote a $6^{\prime}$ eye level and the scale of the first concentric ring segment were to be $2^{\prime}$ this rectangle would have a 2 to 6 proportion. To establish the horizontals which correspond to the other concentric rings, one establishes a $1 / n$ series of distances below eye level where ( $n$ ) is the vertical distance from the eye level to the horizontal which corresponds to the closest ring. Thus, the second ring will be one-half as far from the eye level as the first; the third ring, one-third; fourth ring, onefourth, etc. One can see that this is so when one considers that objects of the same intrinsic size will appear one-half as large when they are twice as for away, onethird as large when three times as far away, etc.

Now that we have the grid established, it can be easily seen that the locations of various control points may be located in an azimuthal sense with reference to the vertical lines of the grid. The vertical displacement of the forms may be similarly located with reference to the horizontal lines of the grid. In the case of the 6' eye level we can see that anywhere within the space there is a standard 6' dimension between any point on the


"ground" surface and the eye level which in turn enables us to establish the vertical scale for any point.

We have found that the best dimensions for the drawings that result from these projections are made to fit within a cylinder approximately $8^{\prime}$ in diameter. This allows an observer to move sufficiently far away from the drawing to get into a comfortable visual range. By constructing these drawings on the normal $36^{\prime \prime}$ paper one has sufficient vertical dimension at this scale to fall comfortably within this visual range.

One final construction which has not been included here-which perhaps would greatly simplify this process - is to be found when one considers that in every case with this kind of drawing an actually straight line would plot as a segment of a sine curve. The maximum point on this sine curve (i.e. for sine of $90^{\circ}$ ) being the closest approach to the particular straight line to the observer. The points of intersection (vanishing points) of the sine curve are on the eye level and are to be found along a radial trace on the plan drawn parallel to the straight line. The amplitude of the sine curve at the maximum point mentioned above is equal to the scale distance of the line in question above or below eye level at the point of closest approach.

Drawing above by Reginald Cude Sophomore, School of Design


Plan: Piazza San Marco, Venice. Perspective below.



STOCK EXCHANGE, MEXICO CITY. DESIGNED AND BUILT BY FELIX CANDELLA

# TOWARD A NEW PHILOSOPHY OF STRUCTURES (Cont.) 

## by Felix Candella

This is a completion of a paper
first appearing in student publications Vol. 5, No. 3

Translated from the Spanish by Dr. G. Poland, Dept. of Modern Languages,
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Having in mind that the end result of the hypothesis is the sketching or simplitication of the physical properties of materials, with the purpose of making attainable the analysis of the structures built with them (materials), the incongruency of the elastic methods for the analysis of three-dimensional laminar structures (shells), whose specific material is reinforced concrete, becomes more evident since the process of obtaining it makes it possible to shape it and adapt it to any surface. As the classic method of attacking this problem gives way to extremely complicated solutions, it results, paradoxically, that the simplifying hypothesis only serve to complicate matters.

When the thickness of the shell is small in relation with other dimensions and radius of curvature, when the loads are uniform and continuous and, above all, if the displacements of the points of the shell when loaded are small in comparison with the thickness, it is possible to apply the theory of membrane stresses. This theory presupposes that all the stresses are tangent to the shell and are distributed uniformly in its thickness, that is to say, that no bending deflection exists that the shell would be incapable of resisting. The six components of the stresses on each point are reduced to three: two normal components and one tangential, or to say it another way, the stress is planar.

If the method of support of the shell allows it to be considered isostatic, equations of static membrane balance are enough to solve the problem, but precisely because of this, it is necessary to fulfill the last condition which requires that the displacements be small in order that these equilibrium equations may remain valid after the deformation. In each case it is necessary to check for the fulfillment of this condition, but since the analytical test is problematical, there is no other recourse but to experiment on models or on actual structures where if necessary, rigid reinforcements which maintain the deformation within acceptable limits can be arranged.

Do not believe, however, that the statement of membrane equations is a simple thing, except in the case of surfaces whose analytical expression is elemental and at any rate, the integration of such equations always entails the appearance of constants of integration whose values can only be determined in certain cases by shape or support conditions.

But in general, the mathematical problem is indeterminate, mainly when the shell is joined to other deformable structural elements (eave beams, other analogous shells, etc.). In these hyperstatic cases balance membrane equations are not enough, and it is usually necessary to use work theorems or virtual work theorems to obtain the necessary number of equations, with which the previously mentioned doubts are raised again, with regard to the deformations and their proportionality with the stresses, now aggravated by the complexity of the problem.

One method for solving them in long cylindrical shells is to consider as unknown hyperstatics the imaginary stresses that would be capable of closing the cracks which would be produced between two contiguous structural elements if the latter were capable of deforming independently. For those who have been interested in these matters, the enormous difficulty of this problem or better yet its almost impossible solution is only too well-known. Its statement expresses the loads by means of "Fourier" series and uses variable complete functions to solve the eighth order differential equation which results from the compatibility condition in spite of important and not always acceptable simplifications. As a consequence of it, it is impossible to keep in mind during the whole mathematical process the physical reality of the problem, and we do not realize the frequent errors until the end of the laborious process which often, for this reason, has to be begun again several times.

Until now, moreover, the analytical problem has only been able to be set up in certain simple cases, those externally isostatic. For example, in long cylindrical shells, even if the expression of the cylindrical surface in cylindrical coordinates is very simple by this method. Only those shells simply supported can be analyzed. Those which are continuous over several supports are outside the field of application of this method as are the majority of cases arising in practice. One would have to ignore the continuity which is one of the greatest advantages of reinforced concrete, in these cases.

If we keep in mind the indeterminacy inherent in deformations in reinforced concrete before knowing the quantity and the position of the reinforcing steel, the only certainty that the method outlined gives us is that of the extraordinary complexity of the mathematical process. For many minds this complication is synonymous with exactness, but the reality of such exactness is completely. illusionary. The proof is that in measuring and placing the reinforcing steel, a criterion totally opposed to that which has been maintained during all the analytical development is followed. Although the method of measuring is not usually mentioned in technical literature dealing with these matters - perhaps in order not to render its incongruency too evident - the truth is that it is done in a rather irrational method. According to Johansen ${ }^{36}$ "The tension stresses are added to a total stress and the reinforcement area is determined dividing this stress by the work coefficient, ignoring the fact that the deformations and the work coefficient do not correspond to each other since the deformations are not constant throughout the zone submitted to tension streses. With these serious discrepancies between stresses and deformations, the justification for all the mathematical work and complicated calculations disappear, since its basis is the theory of elasticity in which stress and deformation are related.

The immediate consequence of the foregoing considerations is that methods based on the theory of elasticity are not appropriate for the analysis of hyperstatic structures of reinforced concrete. As these methods are the only ones permitted by the majority of codes, we find ourselves faced with the unusual fact that we cannot apply to the calculation of structures of reinforced concrete - nearly the only structural material - methods that are in agreement with its characteristics.

To those who consider this statement an exaggeration, we recommend that they simply try to determine analytically, as an illustrative problem, the deformation or elastic limits of a reinforced concrete beam simply supported on its ends, and, after analyzing this simple problem conscientiously, that they reconsider whether the hypotheses that the turn of a section is proportional to the moment that causes it is admissable or if they believe that they can establish seriously the problem of matching the deformations of the long cylindrical shell and its edge beam.

The matter is not, however, as serious as it appears because reinforced concrete, even if it does not have the conditions that elasticity attributes to its, possesses, on the other hand, other characteristics which added to a conservative, although not an explicit, safety coefficient, contribute effectively to the stability of the structures raised with it. We are referring to the plastic deformations that materials are capable of allowing when the stresses reach determined limits.

Although our intention in this work is only to set forth the problem once again, we believe it necessary, so that we may not be censored as pessimists or as ones not doing constructive work, to review the bases upon which the solution has been attempted, hoping that it, together with the bibliography on subject matter which we add, will serve as a stimulus for cooperation that we consider urgently needed to lead to the establishment and development of a new structural theory that can be admitted generally as a substitute for the one already in use.

Let us then attempt to express as briefly as possible, the way in which the plasticity preceeding the break contributes, by means of a change of form, in helping those parts less loaded to help those more loaded and cooperates in the advantageous use of the continuity of the structure.

Let us examine in the first place what happens in a section of reinforced concrete subjected to simple flexure and reinforced only on the tension traction side so that the strains increase until reaching the breaking point (Figure 3). When they are small, the section functions almost as if it were homogeneous and the distribution of stresses were triangular in the compression zone as well as in the tension zone*. As the bending moment increases, the resistance capacity to tension is exceeded after passing through a brief plastic period in which the lower part of the stress diagram curves; the section cracks and tensions are permitted only by reinforcing, raising the position of the neutral axis. On increasing the moment again, the stress in the extreme compression fibres reaches the limit of plasticity or of fluidity (Fig. 1), but it does not pass it, except that the deformation upon increasing without increasing the stresses allows (returning to Navier's hypotheses of linear deformation) the whole compression zone to reach the stress limit, with the compression diagram curving until it approximates a rectangle.



Fig. 1 (left above)
DIAGRAM OF DEFORMATIONS IN STRUCTURAL STEEL.

Fig. 2 (right above)
DIAGRAM OF DEFORMATIONS IN CONCRETE.

Fig. 3 (left)
DIAGRAM OF STRESS DISTRIBUTION IN REINFORCED CONCRETE BEAM.

At the same time it may or may not happen that the reinforcement also might reach its limit of fluidity, (Fig. 2) depending on the percentage of steel in relation to the quality of the concrete.** In the event that a plastic deformation of the reinforcement (under-reinforced sections) is produced, the neutral axis rises rapidly, the lever arm of the internal couple increases slightly, and at last failure by compression in the concrete occurs accompanied by large angular deformations. In the second case of strong reinforcement, the breaking of concrete takes place before the steel yields. We see then that breaking is always produced-directly or indirectly-when the resistance of the concrete exceeds compressive pressure. The difference (which as we see is important) is that in the first case deformations are much greater than in the second.

If one stubbornly wishes to continue supposing against all logic that stresses are proportional to deformations up to the moment of breaking, one would have to admit also that inexplicably the resistance of the concrete to compression by bending is very superior to that of breaking of cylindrical prisms or test cylinders.

The matter would seem somewhat futile if we rely solely on the fact that for usual cases, in which small percentages of reinforcing predominate, the final results of the calcu-

[^2]lations of sections by this theory are very similar to those obtained with the usual methods. The importance of the consideration of the plastic state lies in the fact that, on the one hand, it amplifies considerably the working limits of the given amount of reinforcement which a section can admit, ${ }^{11}$ and, above all, that it gives a fuller and more exact knowledge of the true distribution of the internal stresses allowing us to see more clearly the play of stresses and deformations which takes place in the section and to interpret certain experimental results on beams which the theory of elasticity was incapable of explaining satisfactorily.

Moreover, by putting at our disposal a very clear image of the breaking phenomenon it allows us to eliminate the concept of allowable stresses on work coefficients affected by a safety factor which is different for each material. Calculation by the new procedure is made with just one safety factor for the whole section which can be changed at will depending on the greater or lesser probability occurance of load limits.

Its consideration is also fundamental for the subsequent understanding of the process of deformation of complex structures which are so important in determining their own breaking conditions.

We have indicated before that one of the principal advantages of reinforced concrete is its monolithism which produces continuous or redundant structures, but as this continuity complicates the analysis tremendously when classic methods are followed, it is fitting that before beginning the detailed examination of these structures, we completely understand the concept of the role that continuity plays in the resisting functions and above all in the stability of these structures.

This concept is implicit in the very name with which such structures are designated, hyperstatic, statically inderterminate; this name indicates clearly that when the equations of statics are not sufficient to determine the equilibrial conditions of the whole structure, such equilibrium can be achieved by using different methods which depend, in general, on the deformation possibilities of the structure. The support reactions can, therefore, take on very definite values provided that they are compatible with the static equilibrium of the whole and ultimately in not being limited to a unique solution, the structure protects itself from the action of all possible states of load, including faults in the supports, taking on the solution of distribution of stresses and deformations that permits it to resist most appropriately the external forces to which it is subjected in each case.

Such accommodation takes advantage of whatever means it has within its reach to maintain the equilibrium of the structure. One of the most important is the possibility of allowing plastic deformations in the critical sections, that is to say, in those sections which are exposed to limiting stresses before the other sections reach a saturation of stress capacity.

Thus as in statically determinant or isostatic structures, upon the production of this saturation in one of its points or sections, and upon its beginning to yield plasticaly, it draws after it the rest of the structure, tearing it down. In hyperstatic structures in order
for failure to occur, it is necessary that the total number of points or sections that may fail to be greater in number than the degree of indetermination in the structure.

Thus, for example, a beam simply supported on two points will collapse, doubling in two, when any intermediate section yields, but a beam with both ends fixed being hyperstatic of the second order, requires that failure be produced at three of its points-the two embeddings and the center-in order for destruction to occur. If only the ends break, it will remain in the same condition of simple support as the first case. That is to say, the indeterminacy will have disappeared but at the cost of the safety which will now depend on the resistance of a single section. Nevertheless, the deformation of the beam under these conditions will be of the same degree of magnitude-in spite of two of its sections having failed-as those corresponding to the first isostatic beam as long as the remaining section does not reach its elastic limit. Since the failure of the ends does not necessarily imply their breaking but only the fact that this does happen in most cases, by means of plastic deformations (of the same degree of magnitude as customary elastic deformations) upon the material reaching its elastic or fluid limit, it is senseless to make the safety of a structure depend on the possibility that such partial failures may be produced. The safety factor must be applied to the conditions of total breaking, and in this state the distribution of moments or stresses does not have to coincide with the one which determines the theory of elasticity.

In agreement with this theory, in a fixed end beam of constant section and uniformly distributed load, the distribution of moments between the fixed ends and the center is determined in just one way, a value of $\mathrm{pl}^{2} / 12$ corresponding to the first, and of $\mathrm{pl}^{2} / 24$ to the central section.

Let us suppose, in the first place, that it is a steel beam. Upon the elastic limit being exceeded on the ends, they yield plastically, a resisting moment being created which corresponds to such limit. Beginning here, if the loads continue to grow, the moment that the central section has to resist increases until it reaches the value of breaking which will be equal to those of the fixed ends and of $\mathrm{ql}^{2} / 16$, q being the breaking load. That is to say, in the state of breaking, the distribution of moments is produced with the same value in the three critical sections.

This same result could be reached by following the theory of elasticity if one kept in mind the change of the elastic coefficient corresponding to the state of plasticity of the ends, but it is useless to take such pains.

The first semi-articulations will be produced with a load $p=12 / 16 q=0.75 q$, for which reason, if we calculate as usual, with the safety factor of 2 , such semi-articulations will not appear even with the work loads, and we will have obtained a saving of twenty-five per cent.

If the beam is reinforced concrete, the conditions which permit applying elastic theories will hold only as long as the loads are minimum and no section has cracked. Only for
these loads can we consider a distribution of moments with a value twice those in the supports as in the center. Upon the appearance of the first cracks, the elasticity coefficient as well as the moment of inertia diminishes sharply in the corresponding sections. The law of variation of these quantities is, nevertheless, so vague that it makes practically impossible elastic analysis of the variable section beam which results. Fortunately, it is not necessary at all. The only thing that we need to know is the isostatic moment diagram perfectly determined since it only depends on mechanics.

The size of the moment capable of causing semi-articulations depends on the position and amount of the reinforcement. That is to say, it is subject exclusively to our own free will. But varying at will the closing line of the isostatic diagram, we can achieve the distribution of moments that we consider best provided we place the reinforcement in accord with such choice. By means of a deformative process analogous to that described previously for the steel beam, extreme cases of transforming the simply supported concrete beam or two cantilever concrete beams joined at their free ends can be achieved. With the natural restriction, for structures that must remain exposed to weather, that the cracking in the tension zone of the articulations be inadmissible or dangerous. However, in building structures, that are generally covered, such a consideration does not have great importance.

There exists another more important limitation since it prevents great plastic deformations which are the bases of the process. If the percentage of reinforcement in the section in which the semi-articulations are to be produced is superior to the critical, that is to say, if the breaking of the concrete by compression takes place previous to the beginning of the reinforcement's plastic period, the redistribution of moments cannot be produced since unallowable cracks will have been already produced in the aforementioned critical sections.

However, for the average qualities of materials, $\mathrm{Fe}^{c}=120 \mathrm{Kg} / \mathrm{cm}^{2}$ (stress of breaking of concrete cylinders), the percentage limit of the reinforcing is $1.8 \%,{ }^{15}$ sufficiently superior to the usual percentages for which the problem or limitation mentioned is not usually shown, except in exceptional cases in which the height of the beams is very limited and which, in general, are not economical.

Following an analogous process to that which we have explained for the steel beam, one could determine in each particular case the possibility or not that semi-articulation may be produced with work loads.

The analysis of a reinforced concrete beam, continuing over several supports is reduced to keeping in mind the previous considerations, to outlining the diagram of isostatic moments for each beam section and fixing afterwards the closing lines so that they form a continual broken line. Keeping in mind the previously mentioned limitation, we can, for example, make the positive and negative moment equal; provided that we place the reinforcement in accordance with the choice. ${ }^{18}$

This criterion will equally simplify the analysis of more complex structures, such as porticos or multiple frames. It will also make possible the analysis of those structures in


REINFORCED CONCRETE STRUCTURES UTILIZING HYPERBOLIC PARABOLOIDS DESIGNED AND BUILT IN MEXICO BY FELIX CANDELLA.
which the difficulty of the mathematical problem makes usual methods inapplicable. For example, the theory of slab breaking permits the simple analysis of slabs of arbitrary contours with concentrated loads and over pointed supports which the theory of elasticity would be incapable of even planning. The calculation of long cylindrical shells, which, as we indicate, is one of the most complicated problems which confronts the elastic solution is reduced according to theories of breaking, to the calculation of a concrete beam of a section different from the usual ones. That is to say, the calculation is reduced to the problem of equilibrium between stresses and moments which can be solved in a few hours.

Since an insistance on the details of the procedures would prolong this work interminably, and since we would be adding nothing new to that explained by other writers, we suggest to the reader that he consult the included bibliography on these questions.

Perhaps the principal objection to the general acceptance of these methods lies in their extreme simplicity besides the not unimportant requirement that the designer have some criterion. So many are the years of exclusive predominance of the elastic theories with all their mathematical complexities that one may solely rely upon the conviction that these theories constitute the only means of solving the problem.

In this perhaps is found the most serious sin of elasticity. In many instances, its claim to obtaining exact and unique solutions has prevented other solutions and visualizing different approaches to the problem. It is as if, on looking at a building or a sculpture, we were to be furnished only one preconceived fixed point of view limited in such a way as to prevent one from recognizing all the characteristics of the observed object. Undoubtedly, this would give us a restricted and erroneous image of its true condition and dimensions.

The construction of a Gothic cathedral, not withstanding its being carried out without the help of a differential calculus, presupposes an exquisite refinement in the use of a material having such evident limitations as stone, whereas modern technique, overwhelmed, without doubt, by the enormous weight of mathematical science only on rare occasions has been able to approach such subtleties of construction in spite of having a more perfect material, reinforced concrete.

This makes us think that the elastic methods may have constituted an obstruction in the normal development of structural techniques. Although this statement may sound like blasphemy to those many excessively scientific ears, we would like to think for a moment that the progress of such techniques would have been verified by means of an evolution of the intuitive and experimental concepts of the Middle Ages and Renaissance which produced such brilliant results.

It is very possible that in this way, stimulating the ingenuity of builders (for some reason they were named engineers), better use of materials would have been achieved since it would have set forth the problems more frankly without the prejudice of the premise of having to solve them mathematically. The most convenient and logical forms are not generally easy to analyze from a mathematical point of view, and therefore their use has been abandoned in favor of solutions less appropriate but which, nevertheless, allow
simple analysis. It has been forgotten that as Cross says, ${ }^{17}$ "What we need is a structure, not an analysis."

We have come to the aberration of creating-in many cases with enormous difficulties -isostatic conditions articially by means of articulations of structures with evident scorn for the definitive stability of the structures and without other construction justification than to facilitate and analytical setting forth of the problem.

In such a way arches and vaults are only employed in unusual buildings, such as bridges of great spans in which the size of the budget allows devoting an appreciable part to structural calculations; but it would be very difficult for it to occur to no one to use the reduced vault for mezzanines with great open spaces in spite of it being the obvious solution for such a problem.

In summing up, we think we have presented or suggested enough data and reasoning to seriously start thinking about the possibility of substituting more adequate, simpler and more logical procedures for the habitual structural analysis. This would be in accord with the opinion of Van der Broeck, who in the end of his book ${ }^{22}$ says: "Personally I consider the super-emphasis on the elastic analysis of the last fifty years a mistake which shows evidence of losing its preponderance."

The excessive generality of the theme which we set forth and the elemental consideration of the space acceptable in a work of this type has prevented us from adequately plumbing the depths of each of the points sketched. Nor was it our initial idea to exhaust the matter; but it, together with the total absence of complicated formulas, can give rise to an erroneous impression of superficiality which we would regret since it is opposed to our true intention.

Perhaps by analyzing what we have set forth with greater detail, one can estimate that there exists a certain exaggeration in some concepts, but as a starting point toward accepting a new truth, we must necessarily first tear down almost all we have learned previously and according to Ortego y Gasset: "To think is, whether we like it or not, to exaggerate. He who prefers not to exaggerate has to keep quiet; even more, he has to find a way of paralyzing his intellect."

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[^0]:    * Cardellach and Gaudi were contemparary and both residents of Barcelona. The book, "Filosofia de las Estructuras" was published in Barcelona in 1910.

[^1]:    Hangar in Italy (1939). Span: $333 \times 133$ feet with precast elements in reinforced concrete.

[^2]:    ** For greater information on this question see the work already cited by R. Saliger. (15)

